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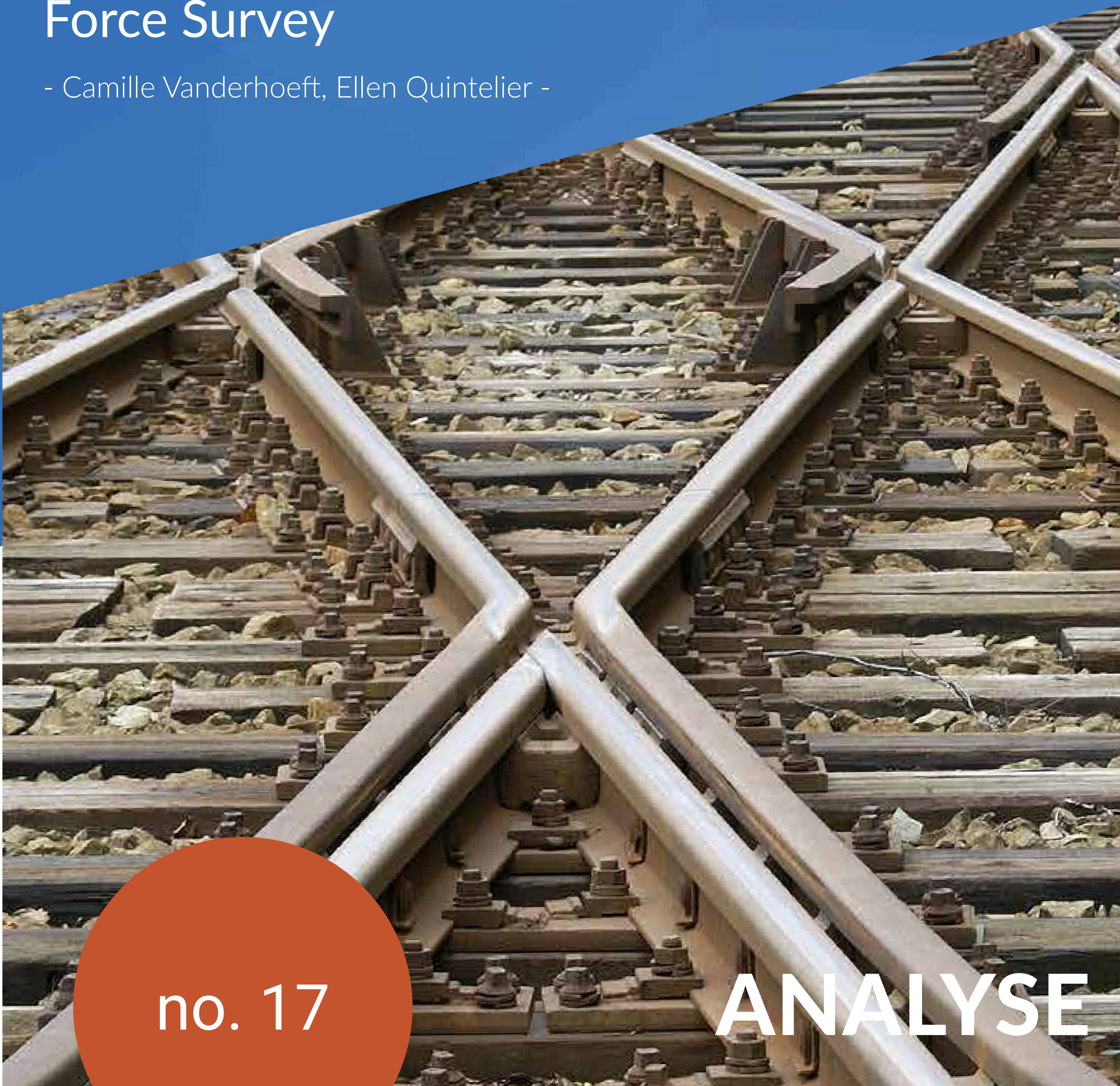
- Camille Vanderhoeft, Ellen Quintelier -



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Estimating ILO labour market transitions based on panel data from the Belgian Labour Force Survey

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ESTIMATING ILO LABOUR MARKET TRANSITIONS BASED ON PANEL DATA FROM THE BELGIAN LABOUR FORCE SURVEY

How many working people remain in employment after one quarter, how many become unemployed or inactive?

What is the evolution in labour market status after one year?

Are there groups in the population that remain more often unemployed?

What methodology does Statbel use for estimating transitions?

Camille Vanderhoeft, Ellen Quintelier

ABSTRACT

In 2017, the Labour Force Survey in Belgium became a panel study. This makes it possible to survey people about their labour market status over an 18-month period, and to determine whether those in employment are still employed three months or one year later, or may have become unemployed or inactive; similarly for the unemployed and inactive. Our objective is therefore to quantify the nine possible transitions between the three labour market statuses (unemployed, employed and inactive). In other words, we intend to estimate 3-by-3 labour market transition matrices.

However, estimating these transition matrices is not as simple as it may seem at first glance: after all, we want the row and column totals of the transition matrices to be consistent with the quarterly or annual figures that can be calculated from the quarterly or annual samples, and for which the results are published as official indicators. We will explain the method in detail in this analysis.

We begin the analysis with an explanation of the structure of the data, and what transitions are possible, in chapter 1. In chapter 2, we will then go into more detail in the methodology, which is entirely based on calibration. In this regard, we took the method used by Eurostat, the European Statistical Office, as our starting point, but we went a step further in order to provide a wide range of breakdowns by various background variables, without losing (too much) coherence with the official indicators. The result of the methodological developments is a fully integrated calibration model. Finally, in chapter 3 we discuss the publication of the estimated transition matrices, and illustrate the difficulties that may arise when transition matrices have to be estimated for smaller sub-populations. With the example of short and long-term unemployed in section 3.4, we show how the transitions in specific sub-populations can be studied.

In the conclusions we summarise the principle findings from our methodological developments.

The focus of this analysis is on the methodology, not on the interpretation, socio-economic explanation or use of the results. A concise discussion of the results can be found on the website of Statbel, the Belgian statistical office, whenever new estimates are published, for example in the form of press releases.

CONTENTS

LIST OF ABBREVIATIONS

LFS	Labour Force Survey
ILO	International Labour Office
AGE	Age group
BQ	Begin quarter, initial quarter
EDU	Educational level
EQ	End quarter, final quarter
IPF	Iterative proportional fitting
LS	Longitudinal sample
NAT	Nationality category
NC	Numerical consistency
NC-C	Numerical consistency according to the classical method
NC-E	Numerical consistency according to the Eurostat method
REG	Region of domicile
RG	Rotation group
SEX	Sex
W	Wave
yyyyQt	Quarter t in year yyyy

LIST OF SCHEMES AND TABLES

1 Introduction

Since the introduction of a panel in the Belgian Labour Force Survey (LFS), not only can we publish quarterly and annual figures, but it is also possible to estimate the transitions in ILO labour market status – i.e. "employed", "unemployed" or "inactive" – (in short: ILO status[3]) from one quarter or year to the next. In this way, it is possible to see how the ILO status of the respondent changes over a number of quarters. The ILO status is by far the most important variable in the Labour Force Survey. This variable indicates whether a person is employed, unemployed or inactive in the reference week according to internationally applied definitions[4]. Transition matrices, which contain the (estimated) numbers of people moving from one ILO status to another, can give an indication of the extent to which unemployed people find work, working people stay employed, etc. After three years of data collection in a panel, there is a limited set of data available that makes it possible to show transitions, and trends in transitions.

In 2017, Statbel, the Belgian statistical office, introduced a rotating panel design for the LFS: every quarter, a new sample or rotation group (RG) is drawn and brought into the field. The panel design is rotating in the sense that every quarter about one fourth of the so-called *quarterly sample* is replaced by a new selection. Respondents in the same RG are interviewed for two consecutive quarters, then not interviewed for two quarters and then interviewed again for the subsequent two quarters (Termote & Depickere, 2018). We refer to this as a 2(2)2 scenario, and the survey of each RG is therefore spread over six quarters. The first time a respondent participates, we refer to the first wave (W1), the second time to the second wave (W2), and so on. Scheme 1 in Termote & Depickere (2018) illustrates this. Since the first quarter of 2018, the quarterly results, based on the quarterly sample, have always been based on data from four waves (for four different RGs); see scheme 2 in Termote & Depickere (2018). Quarterly results in 2017 are based on quarterly samples that are somewhat different in composition; see scheme 4 in Termote & Depickere (2018).

Scheme 4 in Termote & Depickere (2018) also shows that the panel design was already introduced in the third quarter of 2016. However, the first five RGs were surveyed according to scenarios different from the 2(2)2 scenario. The period from the third quarter of 2016 to the fourth quarter of 2017 is a transitional phase, which was necessary to allow the transition from the continuous survey to a panel survey, considering various requirements.

Based on these panel data, we will calculate three types of transitions: *quarterly transitions* (transitions between consecutive quarters), *annual transitions per quarter* (transitions between the same quarters in two consecutive years, also called *quarter-specific annual transitions*) and *annual transitions* (transitions between consecutive years). We will explain these in more detail below.

1.1 Quarterly transitions: transitions between consecutive quarters

Quarterly transitions are based on the overlap of quarterly samples for two consecutive quarters – we refer to the *begin quarter* (BQ) and the *end quarter* (EQ) –, for example, the third and fourth quarters of 2019 (2019Q3 and 2019Q4, respectively), or the fourth quarter of 2019 and the first quarter of 2020 (2019Q4 and 2020Q1, respectively). This overlap, which we will call a *longitudinal sample* (LS) for estimating quarterly transitions, meets a Eurostat requirement of the panel design for the LFS[5]: there must be a minimum theoretical sample overlap of 50% between two consecutive quarterly samples. The Belgian panel design complies with this: the overlap of the quarterly samples for 2019Q3 and 2019Q4, for example, consists of RG13, with observations for each respondent in W3 and W4, and RG17, with observations for each respondent in W1 and W2. Two of the four RGs (i.e. 50% of the original samples) in each of the two quarterly samples therefore give rise to observations in two consecutive quarters.

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[3] With this terminology we indicate that the definitions of the International Labour Office (ILO) are used: concepts such as "employed", "unemployed" and "inactive" should always be interpreted according to the ILO definitions in this analysis.

[4] See (Dutch only) https://statbel.fgov.be/nl/themas/werk-opleiding/arbeidsmarkt/faq.

[5] Regulation 2019/1700 establishing a common framework for European statistics relating to persons and households (https://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX:32019R1700).

Note that each LS for estimating quarterly transitions contains respondents from a well-defined RG with observations in W1 and W2, and respondents from another well-defined RG with observations in W3 and W4. This is an immediate consequence of applying the 2(2)2 scenario.

Results on quarterly transitions were published by Eurostat until the end of 2020 based on its proper methodology. Statbel has now developed its own methodology, inspired by the Eurostat methodology, to quantify transitions itself. From 2021 onwards, Statbel produces and publishes the transition matrices itself, and the previous estimates produced and published by Eurostat will be replaced by Statbel's estimates.

1.2 Annual transitions per quarter: transitions between the same quarters in two consecutive years

Annual transitions per quarter, or *quarter-specific annual transitions*, are based on the overlap of quarterly samples for the same quarter in two consecutive years, e.g. 2018Q2 and 2019Q2, or 2019Q3 and 2020Q3; again, we refer to a BQ and an EQ. This overlap, which we will call a *longitudinal sample* (LS) for estimating annual transitions per quarter, meets another Eurostat requirement related to the panel design for the LFS: there must be a minimum theoretical sample overlap of 20% between quarterly samples for the same quarter in two consecutive years. The Belgian panel design complies with this: the overlap of the quarterly samples for 2018Q2 and 2019Q2, for example, consists of RG11, with observations for each respondent in W2 and W4, and RG12, with observations for each respondent in W1 and W3; thus, two of the four RGs (i.e. 50% of the original samples) in each of the two quarterly samples give rise to observations in BQ and EQ. With this "theoretical" overlap of 50%, the Belgian panel design is therefore well above the required 20%.

Note that each LS for estimating quarter-specific annual transitions contains respondents from a well-defined RG with observations in W1 and W3, and respondents from another well-defined RG with observations for W2 and W4. This is also an immediate consequence of applying the 2(2)2 scenario.

Results on quarter-specific annual transitions are not published by Eurostat, as for some countries the longitudinal sample (LS) is too small in this regard, given the requested theoretical overlap of only 20%. Since in the Belgian LFS there is a theoretical overlap of 50%, Statbel will be able to produce and publish part of the results for these annual transitions.

1.3 Annual transitions: transitions between consecutive years

To obtain the (global) *annual transitions,* we take the average of four quarter-specific annual transitions. Results on annual transitions are published by Eurostat based on its own methodology. Statbel now also produces and publishes figures, using the methodology described in this analysis.

The LFS is subject to non-response and, as a panel survey, to attrition (despite its mandatory nature). On average, 74.4% of the selected persons responded positively to the first interview (figures for surveys conducted in 2019). Among the respondents in the first wave, 87.2% still participate in the second wave, 90.1% in the third and 93.8% in the fourth. Thus, on average, 54.8% of the original sample remains in the fourth wave. Among those who do not participate, not all have refused to participate: the initial address can be wrong, some people moved, the pollsters fail to make contact with the household, etc. Therefore, the overlap between the quarterly samples is in practice smaller than the theoretical 50% discussed above. Furthermore, the overlap is slightly smaller for the annual transitions compared to the quarterly transitions[6].

1.4 Transition matrices

Quarterly transitions between BQ and EQ can be represented in a transition matrix, i.e. a table, as in Scheme 1 below, with on the diagonal (in the grey cells) the numbers of individuals who do not change their ILO status between BQ and EQ[7] (e.g.

[6] For example, the annual transition 2019Q1-2020Q1 counts 12,667 unweighted observations; the quarterly transition 2019Q4-2020Q1 counts 14,507 unweighted observations.

[7] For some respondents, the ILO status can change several times between the observation in the BQ and the observation in the EQ. However, only the statuses in the reference weeks (in BQ and EQ) are registered; intermediate statuses are not noted. It may therefore be the case that a respondent changes status unnoticed, but the registered status in BQ and EQ is the same; such a respondent contributes to the numbers on the diagonal of the transition matrix.

those who are inactive in the reference week in the BQ and in the reference week in the EQ) and in the other cells the numbers of individuals who do change their ILO status (e.g. from employed in the reference week in the BQ to inactive in the reference week in the EQ). The matrix is completed with marginals: (1) the row totals in the last column, reflecting the distribution of ILO status in the BQ; (2) the column totals in the last row, reflecting the distribution of ILO status in the EQ; (3) the total number of persons. The numbers in a transition matrix are estimated numbers of individuals in a particular (sub-)population under study: in Scheme 1 we use the notation $\hat{N}$ (i.e. $\hat{N}_{11}$, …) for the estimates of the actual (sub-)population numbers N (i.e. N_{11}, …).

Scheme 1 General representation of an estimated transition matrix, with marginals

		ILO status in end quarter (EQ)			Total in begin quarter (BQ)
		Unemployed	**Employed**	**Inactive**	
ILO status in begin quarter (BQ)	**Unemployed**	$\hat{N}_{11}$	$\hat{N}_{12}$	$\hat{N}_{13}$	$\hat{N}_{1+}$
	Employed	$\hat{N}_{21}$	$\hat{N}_{22}$	$\hat{N}_{23}$	$\hat{N}_{2+}$
	Inactive	$\hat{N}_{31}$	$\hat{N}_{32}$	$\hat{N}_{33}$	$\hat{N}_{3+}$
Total in end quarter (EQ)		$\hat{N}_{+1}$	$\hat{N}_{+2}$	$\hat{N}_{+3}$	$\hat{N}_{++}$

1.5 Relative transition matrices, or transition percentage matrices

Relative transitions can be obtained by dividing the numbers in the cells per row i by the corresponding row totals and multiplying by 100, i.e. $\hat{p}_{j|i} = 100 \times \hat{N}_{ij}/\hat{N}_{i+}$; the percentage thus obtained is an estimate for the percentage $p_{j|i} = 100 \times N_{ij}/N_{i+}$ of individuals in status i (e.g. unemployed) in the BQ, which end up in status j (e.g. employed) in the EQ. The last row contains the estimated percentage distribution of ILO status in the EQ: $\hat{p}_{+j} = 100 \times \hat{N}_{+j}/\hat{N}_{++}$. A relative transition matrix, hereafter referred to as a *transition percentage matrix*, is presented schematically in Scheme 2.

Scheme 2 General representation of a transition percentage matrix, with marginals

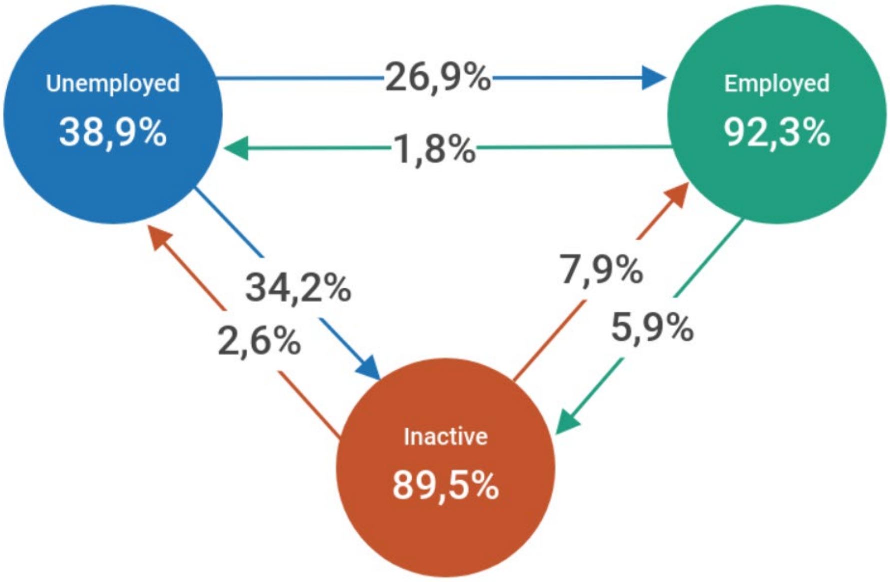
		ILO status in end quarter (EQ)			Total in begin quarter (BQ)
		Unemployed	**Employed**	**Inactive**	
ILO status in begin quarter (BQ)	**Unemployed**	$\hat{p}_{1\|1}$	$\hat{p}_{1\|2}$	$\hat{p}_{1\|3}$	100%
	Employed	$\hat{p}_{2\|1}$	$\hat{p}_{2\|2}$	$\hat{p}_{2\|3}$	100%
	Inactive	$\hat{p}_{3\|1}$	$\hat{p}_{3\|2}$	$\hat{p}_{3\|3}$	100%
Total in end quarter (EQ)		$\hat{p}_{+1}$	$\hat{p}_{+2}$	$\hat{p}_{+3}$	100%

In the Excel files on the Statbel website (see chapter 3), for each transition matrix, the corresponding transition percentage matrix will also be published.

A transition matrix as shown in Scheme 1 can be transformed not only into a transition percentage matrix as shown in Scheme 2, by calculating row percentages $\hat{p}_{j|i}$, but also into an alternative transition percentage matrix by calculating column percentages, namely $\hat{q}_{i|j} = 100 \times \hat{N}_{ij}/\hat{N}_{+j}$. These column percentages $\hat{q}_{i|j}$ indicate what percentage of individuals in status j in the EQ were in status i in the BQ, while row percentages $\hat{p}_{j|i}$ indicate what percentage of individuals in status i in the BQ will be in status j in the EQ. The interpretation of the results in a transition matrix can lead to one or both types of (equivalent) percentages, depending on the user's point of view. For the methodological explanation in this analysis, it does not matter which relative transitions are used; in this text, the term "transition percentage matrix" always refers to row percentages $\hat{p}_{j|i}$, and we only present row percentages in the tables.

In section 1.3 it was stated that an annual transition matrix is obtained as an average of four quarter-specific annual transition matrices (in section 2.9 this is further explored methodologically). Each of the four quarter-specific annual transition matrices is accompanied by a quarter-specific annual transition percentage matrix. It is important to note that the annual transition percentage matrix associated with the annual transition matrix should not be calculated as an average of the four quarter-specific annual transition percentage matrices, but directly from the annual transition matrix.

Transition percentages can be visualised in a diagram, e.g. for the annual transitions between 2019 and 2020:



This diagram shows that, as estimated:

- among the unemployed in 2019, 38.9% are still unemployed in 2020, 26.9% are working in 2020 and 34.2% are inactive in 2020;
- among those working in 2019, 92.3% are still working in 2020, 1.8% are unemployed in 2020 and 5.9% are inactive in 2020;
- among the inactive in 2019, 89.5% are still inactive in 2020, 2.6% are unemployed in 2020 and 7.9% are working in 2020.

1.6 Unweighted transition matrices, or sample size matrices

Finally, each published estimated transition matrix will also be accompanied by an associated "unweighted transition matrix", which is simply the matrix with the numbers of respondents in the LS on which the transition matrix is based: we therefore also refer to a *sample size matrix*. We schematically present a sample size matrix in Scheme 3.

Scheme 3 General representation of a sample size matrix, with marginals

		ILO status in end quarter (EQ)			Total in begin quarter (BQ)
		Unemployed	**Employed**	**Inactive**	
ILO status in begin quarter (EQ)	**Unemployed**	n_{11}	n_{12}	n_{13}	n_{1+}
	Employed	n_{21}	n_{22}	n_{23}	n_{2+}
	Inactive	n_{31}	n_{32}	n_{33}	n_{3+}
Total in end quarter (EQ)		n_{+1}	n_{+2}	n_{+3}	n_{++}

A sample size matrix shows the unweighted distribution of the LS by ILO status in the BQ and in the EQ. The transition matrix shows the weighted distribution of the LS by ILO status in the BQ and in the EQ: each estimate $\widehat{N}_{ij}$ in the transition matrix is the sum of the longitudinal calibrated weights for the corresponding n_{ij} respondents in the LS. The sample size matrix gives indications on the reliability of the estimated transition (percentage) matrix: the larger the number n_{ij}, the more accurate the estimates $\widehat{N}_{ij}$ and $\hat{p}_{i|j}$.

In the Excel files on the Statbel website (see chapter 3), for each transition matrix, the corresponding sample size matrix will also be published.

Note that the sample size matrix associated with an annual transition matrix is simply the sum of the sample size matrices associated with the four quarter-specific annual transition matrices.

Calculating the longitudinal calibrated weights is the central subject of this analysis: see chapter 2.

2 Methodology

The methodology developed by Statbel will be illustrated in this section using the pair of consecutive quarters 2018Q3 and 2018Q4, for which *quarterly transitions* will be estimated. The methodology is the same for estimating *quarter-specific annual transitions*, based on pairs of the same quarters in consecutive years.

Our methodological explanation differs from that of Eurostat (Eurostat, 2015a and Eurostat, 2015b) by rigorously using general calibration methodology, as developed by Deville and Särndal (1992). This also includes suitable general software: Statbel uses the SAS® macro CALMAR2 (Le Guennec and Sautory, 2002 and Sautory, 1993). This allows simpler models (as introduced by Eurostat) to be systematically extended to broaden the objectives without making the calculations more difficult.

Statbel's methodology for estimating quarterly transitions and quarter-specific annual transitions is discussed in detail in sections 2.1 to 2.7. In section 2.8 we compare this methodology with the one introduced by Eurostat (2015b).

In section 2.9 we discuss estimates of (*global*) *annual transitions*, as simple averages of the quarter-specific annual transitions.

2.1 Calibration and calibration variables

Calibration is the process whereby initial weights for the units in a sample are corrected, so that for certain variables the final weighted sample distribution is the same as an (estimated) reference distribution. The sample referred to in this analysis is the *longitudinal sample* (see section 2.2.3), and the units in this sample are individual respondents. The variables considered are referred to as *calibration variables*. The final corrected weights are referred to as *longitudinal calibrated weights*. Some principles and (practical) features of calibration techniques are explained in annex B.

A calibration variable is therefore a variable for which the distribution in the calibrated sample must be the same as an (estimated) reference distribution. The following characteristics of the respondents are potential calibration variables considered in this analysis:

- STAT1, the ILO status in the BQ, and STAT2, the ILO status in the EQ, with values (categories):
  - *Missing* for respondents in age group 0-14
  - *Unemployed*, *Employed* or *Inactive* for respondents in age group 15+;
- SEX, the sex – which is assumed not to change between the BQ and the EQ – with values *Male* and *Female*;
- AGE1, the age group in the BQ, and AGE2, the age group in the EQ, with values (categories) 0-14, 15-24, 25-34, 35-44, 45-54, 55-64, 65-74 and 75+; if necessary, we work with a further grouping of these age groups, e.g. <u>AGE1</u> and <u>AGE2</u> with groups 0-14, 15-34, 35-54, 55-74 and 75+, or <u>AGE1</u> and <u>AGE2</u> with groups 0-14, 15-29, 30-74 and 75+;
- REG1, the region of domicile in the BQ, and REG2, the region of domicile in the EQ, with values *BRU* (Brussels-Capital Region), *VLA* (Flemish Region) and *WAL* (Walloon Region);
- NAT1, the nationality category in the BQ, and NAT2, the nationality category in the EQ, with values (categories) *BE* (Belgian), *EU* (EU28-nationality, excluding Belgium) and *Nt-EU* (not EU28-nationality); if necessary, we work with a further grouping, e.g. <u>NAT1</u> and <u>NAT2</u> with categories *BE* and *Nt-BE*;
- EDU1, the highest level of education attained in the BQ, and EDU2, the highest level of education attained in the EQ, with values (categories) *Low* (no diploma, or at most a lower secondary education diploma), *Middle* (upper secondary education diploma) and *High* (at least a higher education diploma).

Respondents in a sample to be calibrated are excluded if for one or more of these variables the value is undetermined (*missing*). Notice that two types of calibration variables can be distinguished, i.e. background variables (sex, age, region of domicile, nationality and education level) and study variables (ILO status).

Changing the value of a calibration variable between BQ and EQ for some respondents does not pose any problem. For sex (SEX), we did not find any such respondents, but the possibility exists for this variable as well; in the latter case, we would consider variables SEX1 and SEX2. Age group (AGE) inevitably changes for a substantial number of respondents between BQ and EQ. Region of domicile (REG), nationality (NAT) and highest level of education (EDU) change to a lesser extent between

BQ and EQ. Change of ILO status between BQ and EQ is the subject of this study; the variables STAT1 and STAT2 play a special role as calibration variables.

2.2 The samples

For the estimation of transitions between any given pair of quarters, i.e. for the estimation of both quarterly transitions and quarter-specific annual transitions, three samples (of respondents[8]) come into the picture. We discuss these three samples in the following three sub-sections.

2.2.1 The begin quarter sample

This is the sample of respondents in the BQ. This sample contains respondents from four RGs. To estimate quarterly statistics (for core variables), the BQ sample was calibrated to population distributions of various background variables. This provided a calibrated weight w_i^{BQ} for each respondent i in the BQ sample.

For 2018Q3 the BQ sample contains respondents from RGs 8, 9, 12 and 13; see scheme 1 in Termote & Depickere (2018). In the present analysis, this sample is limited to respondents in the age group 15-74.

The BQ sample provides estimated population distributions (for age group 15-74) that will be used as reference distributions in the calibration of the longitudinal sample (LS); this is discussed further in section 2.3.

2.2.2 The end quarter sample

This is the sample of respondents in the EQ. This sample contains respondents from four RGs. To estimate quarterly statistics (for core variables), the EQ sample was calibrated to population distributions of various background variables. This provided a calibrated weight w_i^{EQ} for each respondent i in the EQ sample.

For 2018Q4 the EQ sample contains respondents from RGs 9, 10, 13 and 14; see scheme 1 in Termote & Depickere (2018). In the present analysis, this sample is limited to respondents in the age group 15-74.

The EQ sample provides estimated population distributions (for age group 15-74) that will be used as reference distributions in the calibration of the longitudinal sample (LS); this is discussed further in section 2.3.

2.2.3 The longitudinal sample (LS)

The *longitudinal sample* (LS) is the overlap (or intersection) of the BQ and EQ samples, i.e. the collection of all respondents who were observed in both the BQ and the EQ. In the data files containing the BQ and the EQ samples, the respondents are identified by a unique respondent number, which does not change over the waves. The data file with the LS can therefore easily be constructed by matching the data files with the BQ and the EQ sample on that respondent number. For the pair 2018Q3-2018Q4, this overlap is limited to respondents in two RGs, namely RG9 and RG13; for RG9 we have observations from waves 3 and 4, for RG13 we have observations from waves 1 and 2 (see scheme 1 in Termote & Depickere (2018)). Schematically:

Begin quarter	End quarter	RGs in the overlap	1st RG	2nd RG
			Observations from waves…	
2018Q3	2018Q4	9 and 13	3 and 4	1 and 2

Table B 1 in annex A shows all possible pairs of consecutive quarters since the start of the panel in 2016Q3, with the RGs in the LS, and for each of these RGs the consecutive waves in which respondents in the LS were interviewed. The table was derived from schemes 1 and 4 in Termote & Depickere (2018). Table B 1 shows a standard composition of the LSs from the pair 2017Q4-2018Q1 onward: two RGs, one containing respondents with observations in waves 3 and 4, and the other containing respondents with observations in waves 1 and 2. For earlier pairs, i.e. 2016Q3-2016Q4 to 2017Q3-2017Q4, except for the pair 2017Q1-2017Q2, the composition of the LS deviates from the standard composition: sometimes three RGs are involved and/or observations for respondents may also be available in waves 2 and 3; this is a consequence of the deviating

[8] All samples from this point in the text are samples of respondents; we will not always explicitly repeat this.

scenarios for RGs drawn in the start-up phase of the LFS panel (see scheme 4 in Termote & Depickere (2018)). These deviating compositions could potentially have an impact on the estimated quarterly transitions; see Termote & Depickere (2018).

Table B 2 in annex A shows the composition of LSs for pairs of the same quarters in consecutive years; the table was derived from schemes 1 and 4 in Termote & Depickere (2018). In this table, too, we notice a standard composition from 2017Q1-2018Q1 onward, and deviating compositions for 2016Q3-2017Q3 and 2016Q4-2017Q4, which are due to deviating scenarios for RGs drawn in the start-up phase of the LFS panel (see scheme 4 in Termote & Depickere (2018)).

In this analysis, the LS is always restricted to respondents who are at least 15 and at most 74 years old (i.e. who have not yet turned 75) in both quarterly samples. This avoids a fourth ILO status, i.e. *missing* for 14-year-old people, in the transition matrices.

For the respondents i in an LS, the two calibrated weights w_i^{BQ} and w_i^{EQ} are available; it is the calibrated weight w_i^{EQ} from the EQ that will act as the initial weight in the calibration of the LS and that will therefore be corrected to arrive at the final (longitudinal) calibrated weight for estimating transitions.

2.3 The reference distributions

The BQ and EQ samples are reference samples from which estimates of the reference distributions of ILO status for the calibration of the LS are calculated. The calibrated weight is used for this purpose: w_i^{BQ} for calculating reference distributions from the BQ sample and w_i^{EQ} for calculating reference distributions from the EQ sample. In this way, for example, the estimated distribution of STAT1 (restricted to age group 15-74) in the BQ can be calculated: for each value b of STAT1, the sum of the calibrated weights of the respondents for which STAT1 = b is an estimate of the number of individuals in the population (restricted to age group 15-74) with STAT1 = b, and this estimate is used as a benchmark in the calibration of the LS (if STAT1 is a calibration variable).

For BQ 2018Q3, the estimated population distribution of ILO status (STAT1)[9], which will be one of the main calibration variables, is as follows:

2018Q3	Unemployed	Employed	Inactive	*Total*
Absolute (no. of persons)	300,215.88	4,785,248.91	3,325,058.21	8,410,523.00
Relative (% individuals)	3.57%	56.90%	39.56%	100.00%

For EQ 2018Q4, the estimated population distribution of ILO status (STAT2), which will be one of the main calibration variables, is as follows:

2018Q4	Unemployed	Employed	Inactive	*Total*
Absolute (no. of persons)	290,402.66	4,804,732.96	3,335,187.38	8,430,323.00
Relative (% individuals)	3.44%	56.99%	39.56%	100.00%

Reference distributions of ILO status will also be calculated by sex, by region, … and used in the calibration of LSs. In addition, estimated distributions of background variables (sex, region, age group, etc.) – without the intervention of ILO status – will also be used.

2.4 Basic calibration models, for global transition matrices and transition matrices broken down by sex

The main aim of the calibration of an LS is to make the marginals of estimated ILO status transition matrices consistent with known distributions of ILO status in BQ and EQ. In this section, we will build suitable calibration models step by step and

[9] Statbel does not publish (quarterly or annual) figures for the population of 15-74-year-olds; these figures can be found on the Eurostat website. More information on published figures can be found in chapter 3.

discuss the difficulties involved. In the following sections 2.5, 2.6 and 2.7 these calibration models are extended to achieve additional objectives.

2.4.1 Calibration to global distributions of ILO status

In this sub-section, we aim to adjust the global initial transition matrix for quarter pair 2018Q3-2018Q4 to global estimated distributions of ILO status in BQ (2018Q3) and in EQ (2018Q4).

The LS for the pair 2018Q3-2018Q4 of consecutive quarters contains 13,510 respondents (in age group 15-74 in both quarters), for whom ILO status is known in 2018Q3 (STAT1) and 2018Q4 (STAT2). The unweighted sample transition matrix is shown in Table 1. Note that over 93% of the respondents do not change their ILO status.

Table 1 Unweighted longitudinal sample 2018Q3-2018Q4, by ILO status in BQ and EQ

ILO status	**ILO status 2018Q4**			
2018Q3	Unemployed	Employed	Inactive	Total
Unemployed	224	120	130	474
Employed	57	7,102	296	7,455
Inactive	93	239	5,249	5,581
Total	374	7,461	5,675	13,510

Using the calibrated weights w_i^{EQ} of 2018Q4 for all 13,510 respondents i in the LS, we obtain the initial weighted 3×3 transition matrix, with marginals (i.e. row and column totals, labelled "Total"), in Table 2.

Table 2 Initial transition matrix 2018Q3-2018Q4, and reference distributions

ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	75,566.19	40,185.88	42,546.79	158,298.86	*300,215.88*	*300,922.65*	*300,215.88*
Employed	20,888.66	2,304,490.45	93,986.41	2,419,365.52	*4,785,248.91*	*4,796,514.31*	*4,785,248.91*
Inactive	40,808.23	85,134.34	1,466,478.06	1,592,420.62	*3,325,058.21*	*3,332,886.05*	*3,344,858.21*
Total	137,263.08	2,429,810.66	1,603,011.26	4,170,085.00	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			

The global total of the weights w_i^{EQ} for the respondents in the LS is only 4,170,085.00, which is very low compared to the estimated total population figures (for age group 15-74) 8,410,523.00 in 2018Q3 and 8,430,323.00 in 2018Q4. The reason for this is that the LS covers only two of the four RGs in each quarter, as discussed earlier.

Adjusting the transition matrix in Table 2 to the reference distributions from BQ and EQ, which we also find in Table 2 in the column labelled "Ref. distribution 2018Q3 (a)" and in the row labelled "Ref. distribution 2018Q4", is a calibration of the LS according to a model with the following linear structure:

$$\text{STAT1} + \text{STAT2} \qquad \text{(LS-1)}$$

This calibration cannot be performed immediately because of numerical inconsistency between the reference distributions: the totals are different. In calibration theory, this means that the calibration equations for model LS-1 are numerically inconsistent. This inconsistency can be eliminated in (at least) two ways:

- The *classical method*: the reference distribution "Ref. distribution 2018Q3 (a)" from the BQ is multiplied by the factor 8,430,323.00/8,410,523.00 ≅ 1.002354. We find the corrected distribution in the column labelled "Ref. distribution 2018Q3 (b)" in Table 2; the underlining indicates that the figure in column (a) is adjusted for each ILO status.

- The *Eurostat method*[10]: only the figure for ILO status *Inactive* in the distribution "Ref. distribution 2018Q3 (a)" from the BQ is modified by the difference 8,430,323.00 – 8,410,523.00 = 19,800.00; i.e. 3,325,058.21 is adjusted to 3,325,058.21 + 19,800.00 = 3,344,858.21. We find the corrected distribution in the column labelled "Ref. distribution 2018Q3 (c)" in Table 2; the underlining indicates that only the figure for ILO status *Inactive* is adapted.

Note that the factor 1.002354 in the classical method as well as the modification 19,800.00 in the Eurostat method reflect a (likely) population growth. The methods also work in the event of population shrinkage.

After applying any of the two methods for obtaining numerical consistency – we refer to NC methods or models –, the LS can be calibrated, according to model LS-1, to a corrected reference distribution of ILO status in the BQ (2018Q3) and the reference distribution of ILO status in the EQ (2018Q4). The result of both calibrations for the pair 2018Q3-2018Q4 are presented in Table 3. Note that the marginals of the transition matrix are indeed equal to the corresponding (corrected) reference distributions.

Calibration in accordance with model LS-1 can be performed with an *iterative proportional fitting* or IPF method. This is the method that is applied in Eurostat (2015b) (see section 2.8); see also annex B.5. Statbel applies the more universal Newton-Raphson method, via the SAS® macro CALMAR2, in particular with a view on extensions of the simple model LS-1. To apply CALMAR2, the calibration method or function must be chosen (see annex B). Statbel has chosen the exponential method because it corresponds to the IPF method. In order to keep this alignment between Statbel's and Eurostat's method, Statbel always chooses the exponential method to apply the LS models developed below (via CALMAR2).

Table 3 Estimation of global transitions 2018Q3-2018Q4 with model LS-1, after global correction for numerical inconsistency

After applying the classical method to eliminate numerical inconsistency							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	151,285.33	71,737.93	77,899.39	300,922.65	*300,215.88*	*300,922.65*	*300,215.88*
Employed	46,349.14	4,559,446.13	190,719.04	4,796,514.31	*4,785,248.91*	*4,796,514.31*	*4,785,248.91*
Inactive	92,768.19	173,548.90	3,066,568.96	3,332,886.05	*3,325,058.21*	*3,332,886.05*	*3,344,858.21*
Total	290,402.66	4,804,732.96	3,335,187.38	**8,430,323.00**	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			
After applying the Eurostat method to eliminate numerical inconsistency							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	150,637.85	72,628.75	76,949.28	300,215.88	*300,215.88*	*300,922.65*	*300,215.88*
Employed	45,528.92	4,553,865.52	185,854.46	4,785,248.91	*4,785,248.91*	*4,796,514.31*	*4,785,248.91*
Inactive	94,235.89	178,238.69	3,072,383.63	3,344,858.21	*3,325,058.21*	*3,332,886.05*	*3,344,858.21*
Total	290,402.66	4,804,732.96	3,335,187.38	**8,430,323.00**	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			

The grey background marks figures from the originally calibrated BQ and EQ samples that are reproduced exactly: there are more of them with the Eurostat method than with the classical method. While the classical method does not reproduce the absolute distribution of ILO status in BQ 2018Q3, it does reproduce the relative distribution of ILO status in BQ 2018Q3: indeed, in relative terms, the distributions under "Ref. distribution 2018Q3 (a)" and "Ref. distribution 2018Q3 (b)" are exactly the same. The Eurostat method, on the other hand, reproduces exactly the absolute figures for the unemployed and

[10] We refer to "the Eurostat method" because, according to the Eurostat (2015a) report of the Task Force Flow Statistics, there is a consensus on this method: "*The favoured approach prioritises consistency with the target quarter, i.e. guarantees that the total longitudinal population is identical to the one of the target quarter and the more recent figures of employed, unemployed, and inactive in that quarter are exactly met when adding up the levels of the transition matrix. For the initial quarter this would only be the case for employment and unemployment – inactivity would serve as a residual category, i.e. possible total population differences between the two quarters would be assigned to the inactive population in the initial quarter.*" and "*Similar weighting conditions enforcing consistency with five of the six marginal values are used by several countries producing flow statistics already.*"

employed, but not the relative distribution of ILO status in BQ 2018Q3: in relative terms, the distributions under "Ref. distribution 2018Q3 (a)" and "Ref. distribution 2018Q3 (c)" are different.

The classical and the Eurostat method result in different estimated transition matrices, as shown in Table 3. In absolute terms, the difference is largest for the transition *Inactive-Inactive* (5,814.67 = 3,072,383.63 – 3,066,568.96); in relative terms, the differences are less striking (see Table 4): the largest difference is 0.35 percentage points for the transition *Unemployed-Employed* .

Table 4 Estimation of global relative transitions 2018Q3-2018Q4 with model LS-1, after global corrections for numerical inconsistency

ILO status 2018Q3	*After the classical method*				*After the Eurostat method*			
	ILO status 2018Q4				**ILO status 2018Q4**			
	Unemployed	Employed	Inactive	Total	Unemployed	Employed	Inactive	Total
Unemployed	50.27	23.84	25.89	100.00	50.18	24.19	25.63	100.00
Employed	0.97	95.06	3.98	100.00	0.95	95.16	3.88	100.00
Inactive	2.78	5.21	92.01	100.00	2.82	5.33	91.85	100.00
Total	3.44	56.99	39.56	100.00	3.44	56.99	39.56	100.00
Ref. distribution 2018Q4	3.44	56.99	39.56	100.00	3.44	56.99	39.56	100.00

2.4.2 Calibration to sex-specific distributions of ILO status

We then set out to calibrate the initial transition matrices for men and women to reference distributions of ILO status by sex. Formally, this equates to the application of a calibration model for the LS with the following linear structure:

SEX*(STAT1 + STAT2) = SEX*STAT1 + SEX*STAT2 (LS-2)

For the pair 2018Q3-2018Q4, the initial transition matrices (again using the weights w_i^{EQ}), together with the original reference distributions in the column labelled "Ref. distribution 2018Q3 (a)" and the rows labelled "Ref. distribution 2018Q4", are presented in Table 5. We discuss the columns labelled "Ref. distribution 2018Q3 (b)" and "Ref. distribution 2018Q3 (c)" below, but we already note here that the sums over sex produce the figures in Table 2, except for the column labelled "Ref. distribution 2018Q3 (b)".

Table 5 Initial transition matrices 2018Q3-2018Q4, and reference distributions, by sex

Men							
ILO status 2018Q3	**ILO status 2018Q4**				Ref. distribution 2018Q3		
	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	43,362.60	23,495.56	19,128.01	85,986.16	*170,610.00*	*171,013.58*	*170,610.00*
Employed	11,590.60	1,230,669.29	46,394.42	1,288,654.31	*2,530,132.88*	*2,536,117.96*	*2,530,132.88*
Inactive	21,659.78	41,811.68	649,808.94	713,280.40	*1,494,108.13*	*1,497,642.47*	*1,504,031.13*
Total	76,612.98	1,295,976.53	715,331.37	2,087,920.88	***4,194,851.00***	***4,204,774.00***	***4,204,774.00***
Ref. distribution 2018Q4	*165,517.27*	*2,547,516.81*	*1,491,739.92*	***4,204,774.00***			
Women							
ILO status 2018Q3	**ILO status 2018Q4**				Ref. distribution 2018Q3		
	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	32,203.59	16,690.32	23,418.78	72,312.69	*129,605.88*	*129,909.54*	*129,605.88*
Employed	9,298.06	1,073,821.16	47,591.99	1,130,711.21	*2,255,116.03*	*2,260,399.59*	*2,255,116.03*
Inactive	19,148.45	43,322.65	816,669.12	879,140.22	*1,830,950.09*	*1,835,239.87*	*1,840,827.09*
Total	60,650.10	1,133,834.13	887,679.89	2,082,164.12	***4,215,672.00***	***4,225,549.00***	***4,225,549.00***
Ref. distribution 2018Q4	*124,885.40*	*2,257,216.15*	*1,843,447.46*	***4,225,549.00***			

Again, the reference distributions from the BQ must be adjusted so that the system of calibration equations for model LS-2 is numerically consistent. By sex, we correct the reference distribution of 2018Q3 to the reference distribution of 2018Q4:

- With the classical method: for men, the distribution under "Ref. distribution 2018Q3 (a)" is multiplied by the factor 4,204,774.00/4,194,851.00 ≅ 1.002366, and we find the distribution under "Ref. distribution 2018Q3 (b)"; for women, the distribution under "Ref. distribution 2018Q3 (a)" is multiplied by the factor 4,225,549.00/4,215,672.00 ≅ 1.002343, and we find the distribution under "Ref. distribution 2018Q3 (b)".
- With the Eurostat method: for men, the figure for ILO status *Inactive* in the distribution under "Ref. distribution 2018Q3 (a)" is modified by the difference 4,204,774.00 – 4,194,851.00 = 9,923.00, and we find the distribution under "Ref. distribution 2018Q3 (c)"; for women, the figure for ILO status *Inactive* in the distribution under "Ref. distribution 2018Q3 (a)" is modified by the difference 4,225,549.00 – 4,215,672.00 = 9,877.00, and we find the distribution under "Ref. distribution 2018Q3 (c)".

Calibration model LS-2 can then be applied. This can be done using the IPF method for men and women separately; however, Statbel has chosen to use CALMAR2, with the exponential calibration method, as mentioned in the previous sub-section.

The resulting transition matrices are presented in Table 6, and the resulting relative transition matrices in Table 7. As is the case with the calibration with model LS-1, we find that also in the calibration with model LS-2 the classical and the Eurostat method to obtain numerical consistency do not produce very large differences in the results. The figures on a grey background are figures from the originally calibrated BQ and EQ samples that are reproduced exactly; see also the remarks to Table 3.

Table 6 Estimation of sex-specific transitions 2018Q3-2018Q4 with model LS-2, after corrections by sex for numerical inconsistency

After applying the classical method to achieve numerical inconsistency							
Men							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	90,507.62	43,967.79	36,538.17	171,013.58	*170,610.00*	*171,013.58*	*170,610.00*
Employed	25,397.17	2,417,684.53	93,036.26	2,536,117.96	*2,530,132.88*	*2,536,117.96*	*2,530,132.88*
Inactive	49,612.48	85,864.50	1,362,165.49	1,497,642.47	*1,494,108.13*	*1,497,642.47*	*1,504,031.13*
Total	165,517.27	2,547,516.81	1,491,739.92	**4,204,774.00**	***4,194,851.00***	***4,204,774.00***	***4,204,774.00***
Ref. distribution 2018Q4	*165,517.27*	*2,547,516.81*	*1,491,739.92*	***4,204,774.00***			
Women							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	61,033.08	28,125.93	40,750.53	129,909.54	*129,605.88*	*129.909,54*	*129,605.88*
Employed	20,854.76	2,141,538.41	98,006.42	2,260,399.59	*2,255,116.03*	*2,260,399.59*	*2,255,116.03*
Inactive	42,997.56	87,551.81	1,704,690.50	1,835,239.87	*1,830,950.09*	*1,835,239.87*	*1,840,827.09*
Total	124,885.40	2,257,216.15	1,843,447.46	**4,225,549.00**	***4,215,672.00***	***4,225,549.00***	***4,225,549.00***
Ref. distribution 2018Q4	*124,885.40*	*2,257,216.15*	*1,843,447.46*	***4,225,549.00***			
After applying the Eurostat method to achieve numerical inconsistency							
Men							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	90,185.46	44,455.67	35,968.86	170,610.00	*170,610.00*	*171,013.58*	*170,610.00*
Employed	24,997.79	2,414,666.63	90,468.45	2,530,132.88	*2,530,132.88*	*2,536,117.96*	*2,530,132.88*
Inactive	50,334.01	88,394.51	1,365,302.60	1,504,031.13	*1,494,108.13*	*1,497,642.47*	*1,504,031.13*
Total	165,517.27	2,547,516.81	1,491,739.92	**4,204,774.00**	***4,194,851.00***	***4,204,774.00***	***4,204,774.00***
Ref. distribution 2018Q4	*165,517.27*	*2,547,516.81*	*1,491,739.92*	***4,204,774.00***			
Women							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	60,700.65	28,514.91	40,390.33	129,605.88	*129,605.88*	*129,909.54*	*129,605.88*
Employed	20,433.80	2,138,981.63	95,700.60	2,255,116.03	*2,255,116.03*	*2,260,399.59*	*2,255,116.03*
Inactive	43,750.95	89,719.61	1,707,356.53	1,840,827.09	*1,830,950.09*	*1,835,239.87*	*1,840,827.09*
Total	124,885.40	2,257,216.15	1,843,447.46	4,225,549.00	***4,215,672.00***	***4,225,549.00***	***4,225,549.00***
Ref. distribution 2018Q4	*124,885.40*	*2,257,216.15*	*1,843,447.46*	***4,225,549.00***			

Table 7 Estimation of sex-specific transition percentages 2018Q3-2018Q4 with model LS-2, after corrections by sex for numerical inconsistency

Men								
	After the classical method				*After the Eurostat method*			
ILO status 2018Q3	**ILO status 2018Q4**				**ILO status 2018Q4**			
	Unemployed	Employed	Inactive	Total	Unemployed	In work	Inactive	Total
Unemployed	50.92	25.71	21.37	100.00	52.86	26.06	21.08	100.00
Employed	1.00	95.33	3.67	100.00	0.99	95.44	3.58	100.00
Inactive	3.31	5.73	90.95	100.00	3.35	5.88	90.78	100.00
Total	3.94	60.59	35.48	100.00	3.94	60.59	35.48	100.00
Ref. distribution 2018Q4	3.94	60.59	35.48	100.00	3.94	60.59	35.48	100.00
Women								
	After the classical method				*After the Eurostat method*			
ILO status 2018Q3	**ILO status 2018Q4**				**ILO status 2018Q4**			
	Unemployed	Employed	Inactive	Total	Unemployed	Employed	Inactive	Total
Unemployed	46.98	21.65	31.37	100.00	46.83	22.00	31.16	100.00
Employed	0.92	94.74	4.34	100.00	0.91	94.85	4.24	100.00
Inactive	2.34	4.77	92.9	100.00	2.38	4.87	92.75	100.00
Total	2.96	53.42	43.63	100.00	2.96	53.42	43.63	100.00
Ref. distribution 2018Q4	2.96	53.42	43.63	100.00	2.96	53.42	43.63	100.00

After calibration in accordance with model LS-2, the transition matrices and the reference distributions for men and women can be added up: this results in the global transition matrices and reference distributions in Table 8. We observe:

(1) that the 3×3 transition matrices in Table 8 differ relatively little from the 3×3 transition matrices in Table 3, regardless of the method used to eliminate numerical inconsistency;

(2) that corresponding column totals (in the rows labelled "Total") of these matrices in Table 3 and Table 8 are identically equal. These totals are also the global reference distribution for the EQ 2018Q4;

(3) that the reference distributions under the heading "Ref. distribution 2018Q3 (a)" are all equal. This is the original global reference distribution for the BQ 2018Q3;

(4) that the reference distributions under the heading "Ref. distribution 2018Q3 (c)" are all equal. This is the corrected reference distribution obtained with the Eurostat method for the BQ 2018Q3;

(5) that corresponding row totals (in the columns labelled "Total") of these matrices in Table 3 and Table 8 are identically equal for the Eurostat method;

(6) that the reference distributions under the heading "Ref. distribution 2018Q3 (b)" in Table 3 and Table 8 differ. These are two versions of the corrected reference distributions for the BQ 2018Q3 obtained by the classical method;

(7) that corresponding row totals (in the columns labelled "Total") of these matrices in Table 3 and Table 8 are not the same for the classical method.

The observations in points (2) and (3) are obvious: estimated numbers, both in the BQ and EQ, for men and women separately in any sub-population (e.g. the unemployed) add up to the estimated total number of persons in that sub-population. The observation in point (4) is obvious for the same reason, but also because the adjustment according to the Eurostat method only applies to the inactive, and because the adjustments for inactive men and inactive women add up separately to the global adjustment for inactive persons. The observation in point (5) is directly linked to the one in point (4). The cause of the problem with the classical method mentioned in points (6) and (7) is the application of different correction factors for men and women in order to obtain numerical consistency. This is an argument for choosing the Eurostat method.

The fact that the differences, after applying the classical method, mentioned in points (6) and (7) are small, follows from the fact that the global correction factor (1.002354; see the previous section) differs only slightly from the correction factors by sex (1.002366 for men and 1.002343 for women; see above). For other breakdowns of the transition matrix than the one by sex, larger differences can be expected.

Table 8 Estimation of global transitions 2018Q3-2018Q4 with model LS-2, after corrections by sex for numerical inconsistency

After applying the classical method to eliminate numerical inconsistency							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	151,540.70	72,093.71	77,288.70	300,923.12	*300,215.88*	*300,923.12*	*300,215.88*
Employed	46,251.92	4,559,222.94	191,042.68	4,796,517.55	*4,785,248.91*	*4,796,517.55*	*4,785,248.91*
Inactive	92,610.04	173,416.30	3,066,855.99	3,332,882.33	*3,325,058.21*	*3,332,882.33*	*3,344,858.21*
Total	290,402.66	4,804,732.96	3,335,187.38	**8,430,323.00**	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			
After applying the Eurostat method to eliminate numerical inconsistency							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	150,886.11	72,970.58	76,359.19	300,215.88	*300,215.88*	*300,923.12*	*300,215.88*
Employed	45,431.59	4,553,648.26	186,169.05	4,785,248.91	*4,785,248.91*	*4,796,517.55*	*4,785,248.91*
Inactive	94,084.96	178,114.12	3,072,659.13	3,344,858.21	*3,325,058.21*	*3,332,882.33*	*3,344,858.21*
Total	290,402.66	4,804,732.96	3,335,187.38	**8,430,323.00**	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			

Note that Model LS-2 can also be formulated as follows:

STAT1 + STAT2 + SEX*STAT1 + SEX*STAT2 (LS-2a)

This makes it clear that model LS-2 not only calibrates to two reference distributions of ILO status by sex, but implicitly also to two global reference distributions of ILO status. Model LS-1 calibrates only to two global reference distributions of ILO status. The global reference distributions for the BQ 2018Q3 are not the same for model LS-1 and model LS-2 if the classical method of correcting for numerical inconsistency is applied. As shown above, this is not directly due to the difference between the models LS-1 and LS-2, but to the difference in the prior classical correction for numerical inconsistency. Indeed, for both models we have made the *minimum* necessary correction, i.e.

- a global correction if model LS-1 – i.e. STAT1 + STAT2 – is applied;
- a correction by sex if model LS-2 – i.e. SEX*(STAT1 + STAT2) – is applied.

In order to solve this problem of changes in the global reference distributions for the BQ 2018Q3 when applying the classical method for eliminating numerical inconsistencies, we can also apply the correction by sex if transition matrices are calibrated according to model LS-1. Calibration according to model LS-1, after applying the classical method by sex, results in the estimates and reference distributions in Table 9 (top panel). The estimates of the transitions in Table 9 are very similar to the estimates in Table 3; the differences are attributable to the differences in the correction method – global for Table 3, by sex for Table 9 – which is mainly reflected in the differences in the corrected reference distribution for the BQ 2018Q3. Note that there are no differences if the Eurostat method is applied.

Table 9 Estimation of global transitions 2018Q3-2018Q4 with model LS-1, after corrections by sex for numerical inconsistency

After applying the classical method to eliminate numerical inconsistency							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	151,285.55	71,737.77	77,899.79	300,923.12	*300,215.88*	*300,923.12*	*300,215.88*
Employed	46,349.33	4,559,447.69	190,720.53	4,796,517.55	*4,785,248.91*	*4,796,517.55*	*4,785,248.91*
Inactive	92,767.78	173,547.49	3,066,567.06	3,332,882.33	*3,325,058.21*	*3,332,882.33*	*3,344,858.21*
Total	290,402.66	4,804,732.96	3,335,187.38	**8,430,323.00**	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			
After applying the Eurostat method to eliminate numerical inconsistency							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	150,637.85	72,628.75	76,949.28	300,215.88	*300,215.88*	*300,923.12*	*300,215.88*
Employed	45,528.92	4,553,865.52	185,854.46	4,785,248.91	*4,785,248.91*	*4,796,517.55*	*4,785,248.91*
Inactive	94,235.89	178,238.69	3,072,383.63	3,344,858.21	*3,325,058.21*	*3,332,882.33*	*3,344,858.21*
Total	290,402.66	4,804,732.96	3,335,187.38	**8,430,323.00**	***8,410,523.00***	***8,430,323.00***	***8,430,323.00***
Ref. distribution 2018Q4	*290,402.66*	*4,804,732.96*	*3,335,187.38*	***8,430,323.00***			

2.5 NC methods as calibration models

2.5.1 Introduction

Among the NC methods introduced in section 2.4, i.e. the methods to achieve numerical consistency between reference distributions from BQ and EQ, we differentiate between two categories of methods: the classical methods, which we will call NC-C methods, and the Eurostat methods, which we will call NC-E methods. We illustrated that NC methods are necessary before calibration models – the so-called LS (calibration) models – can be applied to the LS.

In this section, we will elaborate on the NC methods. This is done in the light of the subsequent extension or refinement of the LS calibration models: these models will make more detailed NC methods necessary.

It turns out to be possible and useful to formulate and use the NC methods themselves as calibration models; we then refer to NC (calibration) models, and, if necessary, differentiate between NC-C models and NC-E models. On the one hand, this allows the calculations for obtaining the desired and necessary numerical consistencies between ILO status distributions from the BQ and EQ sample (which are both calibrated) to be made in an efficient and universal manner, so that the (practical) statistician, who wants to apply a well-defined LS model, can quickly ensure the necessary numerical consistencies. Among other things, this also makes it possible to switch flexibly between LS models, with the aim of comparing multiple LS models and choosing a final model. On the other hand, it also makes it possible to present a clear, formal description of the methods or models, the comparison of models and the choice of a final model.

It appears intuitively that NC-C methods can be formulated as calibration models; it is then a calibration of the calibrated BQ sample to estimated distributions determined entirely from the calibrated EQ sample. The formulation of NC-E methods as calibration models is less obvious. This is a calibration of the calibrated BQ sample to combined aggregate information from both the calibrated EQ and the calibrated BQ sample. The resulting NC-E models also have a special characteristic: calibration totals can be negative.

In the following sub-sections, we present the NC methods already applied above as NC calibration models through their linear structure, we discuss potential extensions of these NC models, as well as the possibility that calibration totals can be negative for the class of NC-E models and the resulting choice of calibration method. The latter is illustrated by an example. The mathematical formulation of NC-C and NC-E models is discussed in annex C.

For the general principles and terminology of calibration models, we refer to annex B.

2.5.2 NC-C and NC-E models: variants

In section 2.4 we have already applied the following NC methods or models to correct for numerical inconsistencies:

- to use the classical method to adjust the global distribution of ILO status in the BQ to that in the EQ:

  1 (NC-C-1)

- to use the classical method to adjust the sex-specific distributions of ILO status in the BQ to that in the EQ:

  SEX (NC-C-2)

- to use the Eurostat method to adjust the global distribution of ILO status in the BQ to that in the EQ:

  STAT1 (NC-E-1)

- to use the Eurostat method to adjust the sex-specific distributions of ILO status in the BQ to that in the EQ:

  SEX * STAT1 (NC-E-2)

Model NC-C-1 produces exactly one global correction factor, which is applied to the calibrated weight of each respondent in the BQ sample. Similarly, for model NC-E-1 we can state that one global correction factor also results, which however is only applied to the calibrated weight of each inactive respondent in the BQ sample; the correction factor for all unemployed and employed respondents in the BQ sample is exactly 1.

Model NC-C-2 produces exactly two correction factors, viz one for each sex; one factor is applied to the calibrated weight of each male respondent in the BQ sample, the other to the calibrated weight of each female respondent in the BQ sample. Similarly, for model NC-E-2, we can state that two correction factors result, viz one for each sex; one factor is applied to the calibrated weight of each inactive male respondent in the BQ sample, the other to the calibrated weight of each inactive female respondent in the BQ sample; the correction factor for all unemployed and employed respondents, both male and female, in the BQ sample is exactly 1.

In annex C we show formally and mathematically that not only the NC-C, but also the NC-E methods can be treated as calibration models; which, among other things, explains the above notation via the linear structure. This allows for further refinement and efficient application – provided relevant software is developed – of NC models by including more background variables in these models. This is necessary in view of the extension (see section 2.6) of the calibration models LS-1 and LS-2, if transition matrices have to be broken down by other background variables than (just) sex. The following models also offer the possibility to correct for numerical inconsistencies between reference distributions:

SEX * REG1 * EDU1 * AGE1 * NAT1 (NC-C-3)

(SEX * REG1 * EDU1 * AGE1 * NAT1) * STAT1 (NC-E-3)

Note that NC-C models 1 to 3 and NC-E models 1 to 3 are all "post-stratification" type models. This means that in general (unique) correction factors can be calculated separately per cell in the crossing of all involved variables, which does not require sophisticated software. For NC-E-3, as for NC-E-1 and NC-E-2, the correction factors for unemployed and employed are always 1.

Model NC-C-3 requires that for each cell in the crossing of the calibration variables involved at least one respondent was found in the BQ sample, and that an estimate of the population figure based on the EQ sample can be calculated. If this is not the case, then model NC-C-3 must be "simplified", which can be done, for example, by

(i) regrouping one or more variables;

(ii) refraining from the full crossing of the calibration variables;

(iii) leaving aside one or more calibration variables;

(iv) a combination of (i), (ii) and/or (iii).

Some variants of NC-C-3 include, for example:

SEX * REG1 * EDU1 * AGE1 * NAT1 (NC-C-3a)

SEX + REG1 + EDU1 + AGE1 + NAT1 (NC-C-3b)

SEX * AGE1 + REG1 + NAT1 * EDU1 (NC-C-3c)

SEX * AGE1 + REG1 + NAT1 * EDU1 (NC-C-3d)

whereby AGE1 and AGE1 are groupings of AGE1, and NAT1 is a grouping of NAT1 (see section 2.1).

A possibly necessary simplification of model NC-E-3 can be obtained following the same reasoning, provided that STAT1 is not changed, that the simplification is done within the brackets, and that consequently STAT1 always occurs in the crossing with each retained term within the brackets:

(SEX * REG1 * EDU1 * AGE1 * NAT1) * STAT1 (NC-E-3a)

(SEX + REG1 + EDU1 + AGE1 + NAT1) * STAT1 (NC-E-3b)

(SEX * AGE1 + REG1 + NAT1 * EDU1) * STAT1 (NC-E-3c)

(SEX * AGE1 + REG1 + NAT1 * EDU1) * STAT1 (NC-E-3d)

Statbel has developed useful SAS® macros to easily apply such models together with CALMAR2; see annex B.6.

The choice of an NC-C or NC-E model will ultimately depend on the final choice of the LS model: see section 2.6.

Finally, we note that all NC models are formulated in terms of the background variables and the ILO status in the BQ, which follows from the fact that the BQ sample is calibrated, and therefore the calibrated weights w_i^{BQ} of the respondents in the BQ sample are corrected.

2.5.3 Possibly negative calibration totals and choice of calibration method

2.5.3.1 NC-C models

NC-C models (generally) always lead to positive calibration totals. Indeed, these totals reflect the distribution of the background variables – marginal and/or joint – included in the calibration model, and are (in this analysis) the sums of positive calibrated weights w_i^{EQ} for the respondents i in the EQ sample.[11]

As already mentioned, the variants NC-C-1 to 3 and NC-C-3a of NC-C models are "post-stratification" type models. This means that the choice of the calibration method (linear, exponential, …) does not affect the solution of the correction factors in the system of calibration equations, and that these models are fully determined by their linear structure. With the requirement that for the respondents in the sample to be calibrated (in this case in the BQ sample of respondents) with the same values for all calibration variables – i.e. a "cell" in the complete crossing of the calibration variables – the same correction factor is obtained, this correction factor can be calculated separately for each cell, namely as the ratio of the corresponding calibration total to the sum of the initial weights of the units in the cell in the sample to be calibrated. (This also does not require any sophisticated software.)

For variants of NC-C models which are not post-stratification, e.g. NC-C-3b to d, a calibration method must be selected. We will not go into any more detail here, because this problem is well known in calibration theory, and because ultimately we have not selected an NC-C model, but an NC-E model to solve problems of numerical inconsistency before applying an LS model to the LS.

[11] In a rare practical case, some calibrated weights w_i^{EQ} may be zero, and then calibration totals equal to zero are possible. We ignore this possibility in our explanations.

2.5.3.2 NC-E models

In annex C, in sub-section C.3, we explain Statbel's choice of the linear method when applying NC-E models, as these can lead to negative calibration totals. We illustrate this in the following sub-section 2.5.4.

This implies that even for post-stratification NC-E models, such as NC-E-1 to 3 and NC-E-3a, the calibration method must be well chosen (in order to apply e.g. CALMAR2). Indeed, negative calibration totals can only be used if one or more negative correction factors result from applying an NC-E model. This rules out, for example, the exponential method, as this does not allow negative correction factors; the logit method could be used, provided that a negative lower bound for the correction factors is applied; etc. Ultimately, we always choose the linear method when applying NC-E models, i.e. also for models such as NC-E-3b to d which are not of type post-stratification. The linear method is always applicable when negative calibration totals occur (and in that case, negative correction factors are guaranteed to be found as well). Note that even if all calibration totals are positive, the linear method may result in negative correction factors. In the context of obtaining numerical consistency via NC models, this is not a problem, as these underlying results do not have to be interpreted and published, which on the other hand is the case for the results – the transition matrices – of applying LS models.

As explained above, it is therefore sufficient to present NC-E models through their linear structure, due to the implicit choice of the linear calibration method.

2.5.4 Example: NC-E model with negative calibration totals

Suppose we want to apply post-stratification model NC-E-3a for the pair of quarters 2018Q3-2018Q4 to achieve numerical consistency between BQ and EQ. Table 10 shows all 13 cells $sreanb$ in the crossing[12] of the six calibration variables SEX, REG1, EDU1, AGE1, NAT1 and STAT1 (indexed with respectively s, r, e, a, n and b) for which a negative calibration total $\tilde{T}^{BQ}_{sreanb}$ is obtained (see annex C.1 for the notations). Of course, this is only possible for $b = 3$ (inactive persons). For the first line in Table 10 the following applies:

$$
\begin{aligned}
\tilde{T}^{BQ}_{srean3} &= T^{BQ}_{srean3} + \left(T^{EQ}_{srean} - T^{BQ}_{srean}\right) \\
&= 2307.35 + (12355.88 - 16446.84) \\
&= 2307.35 - 4090.96 \\
&= -1783.61
\end{aligned}
$$

From this, the negative correction factor follows "manually":

$$
g_{srean3} = \tilde{T}^{BQ}_{srean3} \Big/ T^{BQ}_{srean3} = -1783.61 / 2307.35 = -0.77301
$$

by which the calibrated weights w_i^{BQ}, for all $i \in srean3$, are multiplied. The same negative correction factors (for a total of 94 respondents in the 13 cells $srean3$) are also obtained by applying CALMAR2, using the linear method.

[12] The full crossing of SEX, REG1, EDU1, AGE1, NAT1 and STAT1 contains up to 648 non-empty cells. In practice, for pair of quarters 2018Q3-2018Q4, there are only 585 non-empty cells. Sometimes this is (rather) structural: in age group 65-74 ($a = 6$) there are usually no unemployed people ($b = 1$); sometimes this is coincidental (due to relatively small samples), e.g. cell (1,1,2,15-24,2,1) is empty: there are no unemployed male, medium-skilled, 15-24-year-old, non-Belgian respondents in Brussels.

Table 10 Negative calibration totals under model NC-E-3, resulting in negative correction factors

SEX (s)	REG1 (r)	EDU1 (e)	AGE1 (a)	NAT1 (n)	STAT1 (b)	T^{BQ}_{srean3}	T^{BQ}_{srean}	T^{EQ}_{srean}	$\tilde{T}^{BQ}_{srean3}$	g_{srean3}
1	1	2	35-44	1	3	2307.35	16446.84	12355.88	-1783.61	-0.77301
1	1	3	35-44	1	3	966.19	22975.53	20015.91	-1993.42	-2.06317
1	2	3	35-44	1	3	4451.36	167041.99	159310.30	-3280.33	-0.73693
1	2	3	45-54	2	3	1139.72	16865.93	15383.81	-342.40	-0.30042
1	2	3	55-64	2	3	514.89	8689.63	5580.80	-2593.93	-5.03780
1	3	1	25-34	2	3	1134.26	5865.31	3257.67	-1473.38	-1.29898
1	3	2	45-54	2	3	1703.68	16738.28	9267.72	-5766.89	-3.38496
1	3	3	25-34	2	3	1452.33	9950.56	8455.55	-42.68	-0.02939
1	3	3	45-54	1	3	2974.05	77765.55	72537.33	-2254.17	-0.75795
2	1	2	45-54	1	3	1460.67	15947.48	13249.76	-1237.05	-0.84691
2	1	2	45-54	2	3	604.60	5927.60	4395.32	-927.68	-1.53438
2	1	3	35-44	1	3	1876.05	24685.11	20755.26	-2053.80	-1.09475
2	2	3	35-44	1	3	6220.97	210030.97	197009.83	-6800.18	-1.09311

Although NC-E-3a is a post-stratification model, due to the negative calibration totals, the calibration method in CALMAR2 cannot be chosen arbitrarily. For example, if we select the raking ratio method, CALMAR2 does converge, but the calibration equations are not satisfied for the 13 cells in Table 10: for these cells CALMAR2 eventually makes the correction factor g_{srean3} zero (for all other cells the correction factor is correct).

If model NC-E-3b, i.e. (SEX + REG1 + EDU1 + AGE1 + NAT1) * STAT1, is applied for 2018Q3-2018Q4, there are no negative calibration totals: the negative totals $\tilde{T}^{BQ}_{srean3}$ for the 13 cells in Table 10 then become part of the positive totals $\tilde{T}^{BQ}_{s3}$, $\tilde{T}^{BQ}_{r3}$, $\tilde{T}^{BQ}_{e3}$, $\tilde{T}^{BQ}_{a3}$ and $\tilde{T}^{BQ}_{n3}$. CALMAR2 then produces, even with the linear method, no negative correction factors g_{srean3}. If we use the *raking ratio* method, CALMAR2 still converges; the correction factors for $b = 3$ differ relatively little from those resulting from the linear method; for $b = 1$ and 2, as expected, all g_{sreanb} are exactly equal to 1 for both methods.

Nothing rules out the fact that for other pairs of quarters, some calibration totals for a non-post-stratification NC-E model are nevertheless negative. Therefore, we always opt for the linear method when applying an NC-E model.

2.6 Basic calibration models, with break-down by multiple background variables

2.6.1 State of play

In section 2.4 we showed how calibration models can be built if transition matrices only for the total Belgian population and by sex must be consistent with the quarterly figures for ILO status: this resulted in model LS-2 for calibration of the LSs. In section 2.5 we addressed the underlying problem of inconsistency between the BQ and EQ figures for ILO status. The conclusion of these two sections is as follows:

- if (LS-1) STAT1 + STAT2 is applied with the aim of estimating a global transition matrix for which the marginals are consistent with the distribution of ILO status in BQ and EQ, then the calibrated samples for BQ and EQ can be made consistent via a (minimal) model (NC-C-1) 1 or (NC-E-1) STAT1;
- if (LS-2) SEX*(STAT1 + STAT2) is applied with the aim of estimating transition matrices by sex for which the marginals are consistent with the distributions of ILO status in BQ and EQ, then the calibrated samples for BQ and EQ can be made consistent via a (minimal) model (NC-C-2) SEX or (NC-E-2) SEX*STAT1.

Note that it is also possible to combine LS-1 with a (non-minimal) model NC-C-2 or NC-E-2: more NC is then realised between the quarterly samples than is (at least) necessary to make the global transition matrix consistent with the BQ and EQ distribution of ILO status. The realised NC between the quarterly samples is then not fully exploited, and the breakdown of the transition matrix by sex does not reflect the NC by sex between the quarterly samples, in the sense that the sex-specific distributions of ILO status are not reproduced. We can state that models NC-C-2 and NC-E-2 are *overdetermined* to apply LS-1.

On the other hand, it is not possible in general to combine LS-2 with NC-C-1 or NC-E-1: the achieved global NC between the quarterly samples is not sufficient to successfully apply LS-2.

An additional conclusion from section 2.4 is that using NC-E models has advantages over using NC-C models. As such, below we will only work with NC-E models. The specific technical difficulties that may arise with these models were addressed in section 2.5.3.

2.6.2 Extension of objectives and models

As of 2021, Statbel set itself the target of publishing transition matrices for various sub-populations: not only for the entire Belgian population, but also by sex, by region, by education level, by age group and by nationality category. It is preferable that the marginals for each published transition matrix are consistent with previously published quarterly figures for ILO status.

To achieve this objective, the following combination of calibration models can be applied:

(SEX + REGt + EDUt + AGEt + <u>NATt</u>) * (STAT1 + STAT2) (LS-3)

(SEX * REG1 * EDU1 * AGE1 * <u>NAT1</u>) * STAT1 (NC-E-3a)

Here, REGt stands for both REG1 and REG2 (i.e. region of domicile of the respondent in BQ and EQ respectively), as domicile can change between BQ and EQ; idem for EDUt, AGEt and NATt; see section 2.1. The formulation for LS-3 is a concise way of representing the following linear structure:

(SEX + REG1 + EDU1 + AGE1 + <u>NAT1</u>) * STAT1 + (SEX + REG2 + EDU2 + AGE2 + <u>NAT2</u>) * STAT2

since the versions REG1, EDU1, AGE1 and <u>NAT1</u> of the background variables in the BQ are obviously combined with version STAT1 for ILO status in the BQ, and the versions REG2, EDU2, AGE2 and <u>NAT2</u> of the background variables in the EQ are combined with version STAT2 for ILO status in the EQ. Note that – at least in Statbel's experience – to date SEX has not changed its value between BQ and EQ for any respondent.

The term REG1*STAT1 in LS-3 implies that for each region the transition matrix is marginally consistent with the corrected estimated distribution of ILO status in BQ obtained via NC-E-3a, and the term REG2*STAT2 in LS-3 implies that for each region the transition matrix is marginally consistent with the (unchanged) estimated distribution of ILO status in EQ. Similar for the other background variables.[13] This is clearly an extension of model LS-2. Model LS-3 therefore allows the objective to be achieved; model NC-E-3a provides a way of achieving the required NC between estimated ILO status distributions in BQ and EQ.

Table 11 shows the transition matrix by sex after application of the calibration models LS-3 and NC-E-3a. These results are similar to those in the second part of Table 6, which were obtained after applying calibration models (LS-2) SEX*(STAT1 + STAT2) and (NC-E-2) SEX*STAT1. The margins of the transition matrices are the same in both tables, apart from rounding errors. The transition figures themselves differ, which is due to the difference between models LS-3 and LS-2. The largest absolute difference (in absolute value) is 12,732.95 for employed men in both 2018Q3 and 2018Q4; the largest relative difference (in absolute value) is 11.29% for men in employment in 2018Q3 and inactive in 2018Q4. The largest difference between estimated transition percentages is (in absolute value) 1.9 percentage points, for men who are unemployed in 2018Q3 and employed in 2018Q4.

[13] In certain cases, a small perturbation of the LS is necessary to be able to apply an LS model, and thus to achieve the desired consistency. This is further discussed and illustrated in annex D.

Table 11 Estimation of sex-specific transitions 2018Q3-2018Q4 with model LS-3, after corrections for numerical inconsistency with model NC-E-3a

Men							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	89,572.22	47,673.26	33,364.51	170,610.00	*170,610.00*	*171,013.58*	*170,610.00*
Employed	27,518.78	2,401,933.68	100,680.43	2,530,132.89	*2,530,132.88*	*2,536,117.96*	*2,530,132.88*
Inactive	48,426.27	97,909.88	1,357,694.98	1,504,031.13	*1,494,108.13*	*1,497,642.47*	*1,504,031.13*
Total	165,517.27	2,547,516.82	1,491,739.93	**4,204,774.02**	***4,194,851.00***	***4,204,774.00***	***4,204,774.00***
Ref. distribution 2018Q4	*165,517.27*	*2,547,516.81*	*1,491,739.92*	***4,204,774.00***			
Women							
ILO status	**ILO status 2018Q4**				Ref. distribution 2018Q3		
2018Q3	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	59,955.47	30,714.59	38,935.82	129,605.88	*129,605.88*	*129,909.54*	*129,605.88*
Employed	20,804.83	2,134,385.85	99,925.36	2,255,116.03	*2,255,116.03*	*2,260,399.59*	*2,255,116.03*
Inactive	44,125.10	92,115.72	1,704,586.29	1,840,827.10	*1,830,950.09*	*1,835,239.87*	*1,840,827.09*
Total	124,885.40	2,257,216.16	1,843,447.47	**4,225,549.02**	***4,215,672.00***	***4,225,549.00***	***4,225,549.00***
Ref. distribution 2018Q4	*124,885.40*	*2,257,216.15*	*1,843,447.46*	***4,225,549.00***			

Model NC-E-3a (if applicable) ensures that the calibration totals in model LS-3 are consistent, so that LS-3 can be applied. The post-stratification model NC-E-3a can be applied without sophisticated software but is in a sense overdetermined for application of LS-3. Indeed, it not only guarantees the NC between the marginal distributions of ILO status in BQ and EQ by sex, by region, … separately, but also for any possible combination of the five background variables. Such a detailed NC is only necessary if we would like to apply the following combination of models:

(SEX * REG1 * EDU1 * AGE1 * NAT1) * STAT1 (NC-E-3a)

(SEX * REGt * EDUt * AGEt * NATt) * (STAT1 + STAT2) (LS-3a)

which would be necessary if we wanted to produce a transition matrix for each combination of SEX, REG, EDU, AGE and NAT for which the marginals are consistent with the previously published quarterly figures for ILO status. However, for the pair 2018Q3-2018Q4, model LS-3a is not applicable, in se because the LS is too small; specifically:

- for the crossing SEX * REG1 * EDU1 * AGE1 * NAT1 * STAT1 there are 37 cells empty in the LS, but not in the BQ sample;
- for the crossing SEX * REG2 * EDU2 * AGE2 * NAT2 * STAT2 there are 45 cells empty in the LS, but not in the EQ sample.

The following model combination is sufficient to achieve the originally stated objective:

(SEX + REG1 + EDU1 + AGE1 + NAT1) * STAT1 (NC-E-3b)

(SEX + REGt + EDUt + AGEt + NATt) * (STAT1 + STAT2) (LS-3)

because for model LS-3, NC-E-3b is the minimum model to realise NC between BQ and EQ. This model combination also leads to Table 11, which consequently shows that NC-E-3a is overdetermined for LS-3.

We will add one more term to LS-3, in order to arrive at a final model combination that will be applied to obtain the published transition matrices. This is addressed in the following section 2.7.

2.7 Final calibration model, with addition of (a) structure term(s)

Although the objectives are fully achieved with model LS-3, we add another term SEX*AGE2*REG2 to arrive at model LS-4:

(SEX * REG1 * EDU1 * AGE1 * NAT1) * STAT1 (NC-E-3a)

SEX*AGE2*REG2 + (SEX + REGt + EDUt + AGEt + NATt) * (STAT1 + STAT2) (LS-4)

Adding the term SEX*AGE2*REG2 does not change the choice of the NC-E model (we will discuss below why NC-E-3a, and not NC-E-3b, was ultimately chosen). It is ultimately this model combination that is used by Statbel to produce the transition matrices.

Table 12 shows the transition matrix by sex after application of the calibration models LS-4 and NC-E-3a.[14] These results are similar to those in Table 11, which were obtained after applying calibration models LS-3 and NC-E-3a. The margins of the transition matrices are the same in both tables, apart from rounding errors. The transition figures themselves differ relatively little: the largest absolute difference (in absolute value) is 932.35 for men who are inactive in both 2018Q3 and 2018Q4; the largest relative difference (in absolute value) is 1.25% for men who are unemployed in 2018Q3 and inactive in 2018Q4. The largest difference between estimated transition probabilities is (in absolute value) 0.3 percentage points, for men who are unemployed in both 2018T3 and 2018Q4. The added term SEX*AGE2*REG2 in LS-4 compared to LS-3 therefore only has a limited effect on the transition matrices per sex.

Table 12 Estimation of sex-specific transitions 2018Q3-2018Q4 with model LS-4, after corrections for numerical inconsistency with model NC-E-3a

Men							
ILO status 2018Q3	**ILO status 2018Q4**				Ref. distribution 2018Q3		
	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	90,155.76	47,506.15	32,948.09	170,610.00	*170,610.00*	*171,013.58*	*170,610.00*
Employed	27,368.15	2,402,600.26	100,164.51	2,530,132.92	*2,530,132.88*	*2,536,117.96*	*2,530,132.88*
Inactive	47,993.36	97,410.44	1,358,627.33	1,504,031.14	*1,494,108.13*	*1,497,642.47*	*1,504,031.13*
Total	165,517.27	2,547,516.85	1,491,739.93	**4,204,774.05**	***4,194,851.00***	***4,204,774.00***	***4,204,774.00***
Ref. distribution 2018Q4	*165,517.27*	*2,547,516.81*	*1,491,739.92*	***4,204,774.00***			
Women							
ILO status 2018Q3	**ILO status 2018Q4**				Ref. distribution 2018Q3		
	Unemployed	Employed	Inactive	Total	(a)	(b)	(c)
Unemployed	60,299.26	30,824.17	38,482.45	129,605.89	*129,605.88*	*129,909.54*	*129,605.88*
Employed	20,841.99	2,133,679.02	100,595.03	2,255,116.05	*2,255,116.03*	*2,260,399.59*	*2,255,116.03*
Inactive	43,744.14	92,712.98	1,704,370.01	1,840,827.13	*1,830,950.09*	*1,835,239.87*	*1,840,827.09*
Total	124,885.40	2,257,216.17	1,843,447.49	**4,225,549.06**	***4,215,672.00***	***4,225,549.00***	***4,225,549.00***
Ref. distribution 2018Q4	*124,885.40*	*2,257,216.15*	*1,843,447.46*	***4,225,549.00***			

Why then use the extra term? The meaning of the term SEX*AGE2*REG2 is that the composition or structure according to the variables SEX, AGE2 and REG2 of the population of 15-74-year-olds, as estimated in the EQ, is introduced in the calibrated LS. Of course, the term (SEX + REG2 + EDU2 + AGE2 + NAT2) * STAT2 already does this to a certain extent, as it implies, inter alia, the terms SEX, AGE2 and REG2, which introduces the marginal distributions of these variables in the LS. The term SEX*AGE2*REG2 further adds the joint distributions (two by two, and for all three variables together).

The extra term – which we will call a *structure term* – is inspired by the techniques described by Eurostat (2015b); we will come back to this in section 2.8.

After applying any model combination, the transition matrices can be broken down in many ways. For example, we can break it down by province (NUTS2 level; variable PROV2 similarly to REG2) even if only region (NUTS1 level; variable REG2) is in the model combination (e.g. LS-4 with NC-E-3a), or even if region is not in the model combination (e.g. LS-2 with NC-E-2). Another example: after applying model combination LS-4 with NC-E-3a – the final model currently used by Statbel for publications – transition matrices can be made for any combination of SEX and AGE2. The marginals in these transition matrices will generally not be consistent with the quarterly figures, but the structure term may make the marginals more consistent with the quarterly figures. The ideal method of making the marginals in the cells of the crossing SEX×AGE2 consistent with the

[14] Like model LS-3, model LS-4 (for 2018Q3-2018Q4) also requires perturbation of the LS. See footnote 13 and annex D.

quarterly figures is to extend model LS-4 to SEX*AGE2*REG2 + (SEX + REGt + EDUt + AGEt + NATt + **SEX*AGEt**) * (STAT1 + STAT2); however, such extensions can lead to problems due to a too small LS, as illustrated by the unsuccessful application of model combination LS-3a with NC-E-3a.

Of course, SEX*AGE2*REG2 is not the only potentially useful structure term; other examples include SEX*AGE2, SEX*REG2, AGE2*REG2, REG2*NAT2, PROV2, SEX*PROV2, PROV2*AGE2*NAT2, etc. Statbel's final choice for structure term SEX*AGE2*REG2 is quite arbitrary; further study may result in "more optimal" structure terms.

Another possible effect of a structure term is the reduction of variance, i.e. an increase in the precision of the estimated transition figures. This aspect deserves further investigation and goes hand in hand with determining "optimal" structure terms, and consequently determining "optimal" LS models for producing transition matrices.

Finally, we note that Statbel, together with LS-4, did not choose the more simple but adequate model NC-E-3b, but rather the more detailed model NC-E-3a, to estimate and publish the transition matrices, because at the time of the production and publication of the transition matrices, the software (in SAS®, with CALMAR2 and Statbel's own generic macros) for applying models such as NC-E-3b was not yet ready. The transition matrices in Table 12 can be found on the Statbel website in the downloadable Excel file LFS_TRANSITION_ENG_QQ_P.xlsx, in worksheet 2018Q3-Q4, lines 16 to 28. The same Excel file contains numerous other transition matrices: for other pairs of quarters, and for other breakdowns. A similar Excel file LFS_TRANSITION_ENG_JQ_P.xlsx contains the quarter-specific annual transitions, which are also estimated using the model combination NC-E-3a and LS-4.

2.8 Eurostat's step-by-step approach, and comparison with Statbel's method

In this section we discuss Eurostat's methodology (Eurostat, 2015b). This is merely a brief discussion using the above terminology, to make the similarities/differences with Statbel's final model clear.

Using the notation of this analysis (see annex C.1 and section 2.5.4) Eurostat (2015b) uses the background variables

- SEX ("sex"), for which we indicate the categories with the index s, and
- AGE2 ("10-year age group", with categories 15-24, 25-34, … 65-74), for which we indicate the categories with the index $\bar{a}$ (we used index a for the categories of variable AGE1),

and the study variables

- STAT1 (ILO status in the BQ), for which we indicate the categories with the index b, and
- STAT2 (ILO status in the EQ), for which we indicate the categories with the index $\bar{b}$.

For a given pair of quarters, all persons i in the LS have an initial weight w_i^{EQ}. From this, Eurostat (2015b) calculates an initial transition matrix for each of the 12 combinations $s\bar{a}$ of SEX and AGE2. Each of these transition matrices is corrected for the estimated distribution of ILO status in the EQ (STAT2) for the sub-population $s\bar{a}$. This first correction corresponds in our modelling approach to a calibration of the full LS according to the post-stratification model

$$(\text{SEX} * \text{AGE2}) * \text{STAT2} \qquad \text{(LS-0a)}$$

We can denote the correction factors as $c_{s\bar{a}\bar{b}}$; the result of this first correction is a new weight $c_{s\bar{a}\bar{b}}w_i^{EQ}$ for each person i in the LS. This results in an adjusted transition matrix for each combination $s\bar{a}$, based on the new weights $c_{s\bar{a}\bar{b}}w_i^{EQ}$.

A second correction of the LS is prepared by Eurostat (2015b) as follows:

- The adjusted transition matrices are summed across age groups $\bar{a}$ for each sex s, which produces two sex-specific adjusted transition matrices.
- For each sex s and for the full age group 15-74, the estimated distribution of ILO status in the BQ (STAT1) is adjusted to be consistent with the distribution of ILO status in the EQ (STAT2). This is done by adjusting (by sex s) the estimated number of inactive persons in the BQ so that (by sex s) the sums of the estimated numbers of employed, unemployed and inactive persons in the BQ and EQ are equal. In our modelling approach, this adjustment corresponds to a calibration of the BQ sample of 15-74-year-olds, each having a weight w_i^{BQ}, according to the NC-E model

SEX * STAT1 (NC-E-0)

For each person i in the BQ sample this produces a new weight $g_{sb} w_i^{BQ}$. Note that the correction factors g_{sb} are equal to 1 for $b = 1$ (employed) and $b = 2$ (unemployed). Note too that model NC-E-0 is the same as model NC-E-2 (see section 2.5.2).

Eurostat (2015b) then uses the IPF method to correct the two sex-specific adjusted transition matrices to the corresponding distributions (by sex s) of ILO status in BQ and EQ. This second correction corresponds in our modelling approach to a calibration of the full LS according to the model

SEX * (STAT1 + STAT2) (LS-0b)

We denote the correction factors as $\acute{c}_{sb\bar{b}}$; the result of this second correction is the final weight $\acute{c}_{sb\bar{b}} c_{s\bar{a}\bar{b}} w_i^{EQ}$ for each person i in the LS. Note that model LS-0b is the same as model LS-2 (see section 2.4.2).

It is noteworthy that in Eurostat's method the processing of the BQ sample according to model NC-E-0 can come before the two successive corrections of transition matrices without any problem, we can therefore conclude that Eurostat's method in Statbel's approach corresponds to

- applying model NC-E-0 (or NC-E-2) to the BQ sample, with initial weight w_i^{BQ};
- then applying model LS-0a to the LS, with initial weights w_i^{EQ}, which results in weights $c_{s\bar{a}\bar{b}} w_i^{EQ}$;
- then applying model LS-0b (or LS-2) to the LS, with new initial weights $c_{s\bar{a}\bar{b}} w_i^{EQ}$, which results in final weights $\acute{c}_{sb\bar{b}} c_{s\bar{a}\bar{b}} w_i^{EQ}$;

The result is the LS sample with calibrated weights $\acute{c}_{sb\bar{b}} c_{s\bar{a}\bar{b}} w_i^{EQ}$, with which final transition matrices can be calculated.

Note that the calculations for the models NC-E-0, LS-0a and LS-0b do not require sophisticated software: NC-E-0 and LS-0a are post-stratification models for which a correction factor per cell in the crossing of the variables in question can be calculated, and LS-0b can be solved with IPF (as usual per value of SEX in two dimensions in accordance with the terms STAT1 and STAT2, or simultaneously for men and women in two dimensions in accordance with the terms SEX*STAT1 and SEX*STAT2).

In this way, the LS is calibrated in two steps. There is in fact no reason for this, so a valid alternative to Eurostat's method can be formulated as follows:

- applying model NC-E-0 (or NC-E-2) to the BQ sample, with initial weight w_i^{BQ};
- then applying model LS-0c to the LS, with initial weights w_i^{EQ}, in which LC-0c achieves the objectives of LS-0a and LS-0b simultaneously:

(SEX * AGE2) * STAT2 + SEX * (STAT1 + STAT2) (LS-0c)

Model LS-0c transforms the initial weights w_i^{EQ} in calibrated weights $\bar{c}_{s\bar{a}b\bar{b}} w_i^{EQ}$, say.

One disadvantage of the step-by-step application of models LS-0a and LS-0b is that the consistency achieved with LS-0a is generally cancelled out by the application of LS-0b. This means that with the correction factors $\acute{c}_{sb\bar{b}} c_{s\bar{a}\bar{b}}$ the calibration equations of model LS-0a are generally not satisfied. The advantage of model LS-0c, with correction factors $\bar{c}_{s\bar{a}b\bar{b}}$, is that all the calibration equations, both those resulting from model LS-0a and those resulting from model LS-0b, are simultaneously satisfied. The correction factors $\acute{c}_{sb\bar{b}} c_{s\bar{a}\bar{b}}$ and $\bar{c}_{s\bar{a}b\bar{b}}$ are therefore generally not equal.

A technical disadvantage of model LS-0c is that a post-stratification technique clearly cannot be used to apply this model. IPF, on the other hand, can be applied in principle, but not (as is usual) in two dimensions, but in three dimensions – corresponding to the three terms SEX*AGE2*STAT2, SEX*STAT1 and SEX*STAT2 – which requires a more advanced implementation of the IPF method. This disadvantage can easily be worked around by using a generic macro such as CALMAR2, which is written in SAS® code. In CALMAR2, the IPF algorithm cannot be chosen: all calibration models that can be applied by CALMAR2 are solved by a highly universal numerical algorithm based on the Newton-Raphson method.

To apply model LS-0c via CALMAR2, a calibration method needs to be chosen. Since the IPF algorithm leads to the same solution of calibration models as the universal algorithm in CALMAR2 when the exponential method (i.e. a multiplicative calibration function) is used, using the exponential method is the obvious choice. This motivated the decision to solve all LS models in this analysis with the exponential method. This choice is therefore in line with the Eurostat method.

Finally, we can compare LS-0c with Statbel's final model LS-4. In the first instance, we ignore the variables in LS-4 that do not occur in model LS-0c, i.e. we reduce LS-4 to the following simpler model:

SEX*AGE2 + (SEX + AGEt) * (STAT1 + STAT2) (LS-4a)

To understand the similarities and differences between LS-4a and LS-0c, we rewrite[15] LS-0c as:

[SEX*AGE2 + SEX*AGE2*STAT2] + [AGE2*STAT2 + SEX*(STAT1 + STAT2)] (LS-0c)

or as:

[SEX*AGE2*STAT2] + [AGE2*STAT2 + SEX*(STAT1 + STAT2)] (LS-0c)

The difference between the second part [AGE2*STAT2 + SEX*(STAT1 + STAT2)] in LS-0c and the second part (SEX + AGEt) * (STAT1 + STAT2) in LS-4a is a term AGE1*STAT1. Adding this term AGE1*STAT1 in LS-0c would therefore extend the original objective of Eurostat's illustrative paper in a natural way: transition matrices would not only be consistent for men and women with results from BQ and EQ, but also by age group.

The difference between the first part [SEX*AGE2 + SEX*AGE2*STAT2] in LS-0c and the first part SEX*AGE2 in LS-4a is – formally – the term SEX*AGE2*STAT2, but because SEX*AGE2 is implied by SEX*AGE2*STAT2, the actual difference consists of (1°) the term STAT2, (2°) the two-way interactions SEX×STAT2 and AGE2×STAT2 and (3°) the three-way interaction SEX×AGE2×STAT2. Moreover, it is the case that the terms STAT2, SEX×STAT2 and AGE2×STAT2 are implied by the second part in LS-4a, or by the extended second part of LS-0c as suggested in the previous section.

An alternative way of comparing LS-4a and LS-0c with regard to the respective first parts, is to state that the structure term SEX*AGE2 in LS-4a is extended to the structure term SEX*AGE2*STAT2 in LS-0c. This appears to be a "large" extension, but, taking into account also the second part in LS-0c, we find that components of SEX*AGE2*STAT2 are already part of the second part of the suggested extension of LS-0c: the terms STAT2, SEX×STAT2 and AGE2×STAT2 are implied by both parts. In other words, there is an "overlap" between the first and the second part in (the extended version of) LS-0c.

Note that LS-4a, unlike LS-0c, is somewhat easier to interpret: the second part allows the consistency objectives regarding various distributions of the study variables STAT1 and STAT2 to be met, while the first part focuses only on the structure of the calibrated EQ sample (i.e. an estimated structure of the population) via background variables. And the extension of LS-4a to LS-4 is therefore somewhat more transparent: extension of the second part to achieve more coherence, extension of the first part to incorporate more structure of the (estimated) population of 15-74-year-olds in the EQ into the calibrated LS.

2.9 Estimating annual transitions

In sections 2.1 to 2.7 calibration models were developed for estimating quarterly transitions and quarter-specific annual transitions. Both types of transitions can be estimated using the same methodology, as both involve the calibration of an LS which is the intersection of two quarterly samples. Except if influenced by the start-up phase of the panel, these LSs all have the same structure: they consist of respondents from two RGs, for which each time exactly one transition is observed. This can be derived from Table B 1 for the quarterly transitions and from Table B 2 for the quarter-specific annual transitions.

Using scheme 1 in Termote & Depickere (2018) – if we were to draw up a similar table for the global annual transitions, which assumes annual samples for two consecutive years (e.g. 2018 and 2019), and for which the LS is the intersection of those annual samples, then that table would look like this:

[15] Rewriting or reformulating the linear structures of calibration models is based on the hierarchical nature of these structures: a term such as e.g. A*B*C (for category variables A, B and C) always implies the terms A*B, A*C, B*C, A, B, C and 1; see annex B.4.1.

Begin year	End year	RGs in the overlap	1st RG	2nd RG	3rd RG	4th RG	5th RG
			Observations from waves…				
2018	2019	10, 11, 12, 13, 14	2 and 4	1 and 3 2 and 4	1 and 3 2 and 4	1 and 3 2 and 4	1 and 3

The LS for 2018-2019 would involve five (consecutive) RGs, which do not all provide data in the same way: for each respondent from the first and fifth RG exactly one transition is observed (between W2 and W4, or between W1 and W3), but for each respondent from the other three RGs two transitions are observed (between W1 and W3, and between W2 and W4). Such two transitions, i.e. two "observations", for the same respondent cannot be considered independent, which goes against the common application of calibration techniques, in which independence between "observations" is assumed.

Because of this problem, annual transitions are simply estimated as an unweighted average of the estimated quarter-specific annual transitions.

Note that the four estimated quarter-specific annual transitions are not independent statistics (by analogy with the non-independence of observations in the LS for estimating annual transitions discussed above). Estimating the annual transitions as an average does not pose a problem. Variance estimation for annual transitions, however, would have to be realised with specially developed techniques that take into account two observations for a large part of the respondents, but that is beyond the scope of this analysis.

Estimating annual transitions as an unweighted average of quarter-specific annual transitions is analogous to estimating annual key indicators (such as number of unemployed, number of employed, etc.) as unweighted averages of quarterly key indicators. The method of variance estimation for annual key indicators was also tuned to the fact that the majority of respondents contribute to two quarterly key indicators.

3 Published figures

From 2021 onwards, Statbel publishes each quarter – i.e. the *current* quarter – the transition matrices for the current quarter compared to the previous quarter (i.e. the latest *quarterly transitions*, denoted by QQ in the name of the downloadable Excel files), and for the current quarter compared to the same quarter one year earlier (i.e. the most recent *quarter-specific annual transitions*, denoted by JQ). Annually, Statbel will also publish the *annual transitions* (i.e. the averages of the four most recent quarter-specific annual transitions, denoted by JJ). Each published transition matrix is accompanied by a matrix of transition rates (or percentages) and a matrix of respondent sample sizes (or unweighted transitions).

For each of the three types (QQ, JQ and JJ) of transition figures, Statbel publishes the transitions for the entire population of 15-74-year-olds, as well as breakdowns by sex, region, age (younger than 30 vs. at least 30 years), education level (low, medium, high) and nationality (Belgian vs. non-Belgian). Below, we discuss the results for each of the three types of transitions and some aspects that should be considered when using and interpreting them.

Since January 2021, Statbel publishes the quarterly transitions (type QQ) and the quarter-specific annual transitions (type JQ) together with the quarterly results for key indicators. For these transitions, Statbel also provides the time series: from 2017Q1-2017Q2 for type QQ and from 2017Q1-2018Q1 for type JQ. Statbel also publishes the annual transitions (type JJ) since 2021. In the future, these will be published together with the annual figures, i.e. at the end of March.

3.1 Quarterly transitions: transitions between consecutive quarters

Below we briefly explain which figures are published on transitions between consecutive quarters and how these can be interpreted. As was the case in chapter 2 we use the pair of quarters 2018Q3-2018Q4 for illustration purposes. The published quarterly transitions for this pair can be found in worksheet 2018Q3-Q4 in the downloadable Excel file LFS_TRANSITION_ENG_QQ_P.xlsx; the figures for the entire population of 15-74-year-olds are compiled below in Table 13. The figures in panel A of this table are the final estimates of the transitions for the pair of quarters referred to. Note that these are the rounded sums of the estimates by sex, as presented in Table 12, e.g. 90,155.76 + 60,299.26 = 150,455.02 in Table 12 becomes 150,455 in Table 13.

The transitions in panel A of Table 13 indicate how many people are estimated to have made a given transition. For example, we see that 48,210 people changed from employed to unemployed. They lost their job, which they still had in 2018Q3, but are actively looking for another job in 2018Q4. The table also shows, for example, how many people who were previously unemployed or inactive found a job: 78,330 unemployed and 190,123 inactive in 2018Q3 found work in 2018Q4.

The totals for the end quarter 2018Q4, i.e. the estimated absolute distribution of ILO status in 2018Q4 (which is not shown in Table 13), are identical to those resulting from calibration of the quarterly sample for 2018Q4. The totals for the begin quarter 2018Q3, for unemployed (300,216) and employed (4,785,249), are identical to those resulting from calibration of the quarterly sample for 2018Q3; for inactive persons, the total (3,344,858) differs from the result of this calibration (3,325,058), as a result of the necessary correction for numerical inconsistency between BQ and EQ.[16]

Table 13 also shows the transition percentages in panel B. These are row percentages calculated from the transition matrix in panel A (see section 1.5 for the reason for the (arbitrary) choice of *row* percentages in this analysis). In the diagonal from top left to bottom right we find the percentages of people who did not make a transition, who are therefore stable in their status. For example, from 2018Q3 to 2018Q4 94.8% remain employed on the labour market (i.e. 4,536,279 of 4,785,249 people). The cells not on the diagonal contain figures for people who did make a transition: for example, 26.1% change from unemployed in 2018Q3 to employed in 2018Q4 (i.e. 78,330 of 300,216 people). Furthermore, of those who were inactive in 2018Q3, 91.6% are still inactive, 2.7% are unemployed and 5.7% are employed in 2018Q4.

[16] For an overview of tables published by Eurostat, see https://ec.europa.eu/eurostat/web/lfs/data/database; the tables LFSQ_UGAN, LFSQ_EGAN and LFSQ_IGAN allow e.g. the totals 300,216, 4,785,249 and 3,325,058 (for 2018Q3) to be found in thousands, i.e. 300.2 (x1000), 4,785.2 (x1000), and 3,325.1 (x1000).

Table 13 Published quarterly transition matrix for 2018Q3-2018Q4, with corresponding transition percentage and sample size matrices – see publication LFS_TRANSITION_ENG_QQ_P.xlsx

A. Transitions

2018Q4 / 2018Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	150,455	78,330	71,431	300,216
Employed previous Q	48,210	4,536,279	200,760	4,785,249
Inactive previous Q	91,738	190,123	3,062,997	3,344,858

B. Transition percentages

2018Q4 / 2018Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	50.1%	26.1%	23.8%	100.0%
Employed previous Q	1.0%	94.8%	4.2%	100.0%
Inactive previous Q	2.7%	5.7%	91.6%	100.0%

C. Unweighted transitions (respondent sample sizes)

2018Q4 / 2018Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	224	120	130	474
Employed previous Q	57	7,102	296	7,455
Inactive previous Q	93	239	5,249	5,581

Finally, in panel C of Table 13 we find the unweighted number of respondents for each of the nine possible transitions, i.e. the respondent sample sizes. Note that these numbers are also presented in Table 1. As a whole, these are less interesting to interpret directly, but they are important as an indication of the precision of the figures in panel A and panel B: the smaller the unweighted number of respondents, the less precise and reliable the corresponding estimates for absolute transition figures in panel A and relative transition figures in panel B.[17] This is important when interpreting the figures, and especially when we consider fluctuations in time series. They also indicate that breakdowns by various background variables could become problematic, as illustrated in the following sections for breakdowns by nationality and age group.

Breakdown by nationality category

One example where the respondent numbers quickly become small is when broken down by nationality. As indicated in section 2.1 the original intention was to make the transition matrices consistent with quarterly estimates of ILO status for three nationality categories, namely *BE*, *EU* and *Nt-EU*. Table 14 shows a low representation of non-Belgians (*EU* and *Nt-EU* combined) in the LS for 2018Q3-2018Q4: the six non-diagonal cells each contain fewer than 30 respondents, making the estimated transitions and transition percentages in these six cells unreliable. A further differentiation between *EU* and *Nt-EU* would therefore not provide more useful results.

Table 14 Distribution of the longitudinal sub-sample 2018Q3-2018Q4 of non-Belgians (EU and Nt-EU combined) according to ILO status in begin and end quarter

2018Q4 / 2018Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	65	14	24	103
Employed previous Q	8	777	28	813
Inactive previous Q	22	27	466	515

What is illustrated here for 2018Q3-2018Q4 applies to most, if not all, pairs of quarters (see the various worksheets in LFS_TRANSITION_ENG_QQ_P.xlsx). That is the reason why in the calibration models we have only retained the dichotomy *BE* versus *Nt-BE* (i.e. NAT1 and NAT2).

[17] Variance estimation for transition figures and percentages has not yet been realised at the time of writing this analysis.

Breakdown by age group

The breakdown of transition matrices by age group poses a problem similar to that discussed above for the breakdown by nationality category. As indicated in section 2.1 10-year age groups 15-24, 25-34, 35-44, 45-54, 55-64, 65-74 (for the population of 15-74-year-olds) were used in the calibrations for estimating transitions. However, especially for the last two categories, the respondent numbers according to ILO status in BQ and EQ are so small – as shown in Table 15 – that this split was not retained in the publications.

Table 15 Distribution of the longitudinal sub-samples 2018Q3-2018Q4 of 55-64- and 65-74-year-olds according to ILO status in begin and end quarter

Age category 55-64				
2018Q4 / **2018Q3**	**Unemployed current Q**	**Employed current Q**	**Inactive current Q**	**Total**
Unemployed previous Q	33	6	22	61
Employed previous Q	4	1,416	72	1,492
Inactive previous Q	10	36	1137	1,183
Age category 65-74				
2018Q4 / **2018Q3**	**Unemployed current Q**	**Employed current Q**	**Inactive current Q**	**Total**
Unemployed previous Q	0	0	1	1
Employed previous Q	0	69	23	92
Inactive previous Q	1	17	2,164	2,182

In the publications, only the dichotomy 15-29 versus 30-74 is used for the breakdown by age group; only one cell in the two transition matrices is then based on fewer than 30 respondents. The problem of publication of (too many) unreliable figures has this way been worked around, but this does create a problem of inconsistency between the marginals of the transition matrices and previous quarterly estimates for the distributions of ILO status in BQ and EQ.

We illustrate this using the transition matrix 2018Q3-2018Q4 for age group 15-29: see Table 16. The estimated transition matrix, with the row totals in the column labelled "Total", can be found in LFS_TRANSITION_ENG_QQ_P.xlsx in worksheet 2018Q3-Q4 (range A53:E56); in the row labelled "Total" we have also inserted the column totals of the transition matrix. So, in the row labelled "Total", on the one hand we find the distribution of ILO status in 2018Q4 for 15-29-year-olds as estimated on the basis of the LS 2018Q3-2018Q4. In the row labelled "Estimate 2018Q4", on the other hand, we find the distribution of ILO status in 2018Q4 (for 15-29-year-olds) as estimated in that quarter based on the full quarterly sample (see footnote 16 to find these figures on Eurostat's website). These distributions are not equal because in the calibration of the quarterly sample of 2018Q4 an age group has the upper limit 29 (the calibration variable for age has groups 0-4, 5-9, ... 25-**29**, 30-34, ... 70-74, 75+), while for the calibration of the LS 2018Q3-2018Q4 this is not the case (the calibration variable for age then has classes 15-24, 25-34, ... 65-74); note that, on the other hand, 15 is the lower limit of an age group in both calibrations. The rows labelled "Difference" and "% Difference" quantify the discrepancy between both distributions of ILO status for 15-29-year-olds in 2018Q4. The same exercise can be made for the distribution of ILO status in 2018Q3 for 15-29-year-olds. The result can be found in the columns labelled "Estimate 2018Q3", "Difference" and "% Difference". Note that we have deliberately omitted the figures for "Inactive previous Q", as the estimated number of 1,004,312 would have to be corrected to be comparable with the given total of 998,745.[18]

[18] The corrected figure that compares to 998,745 is 983,534 = 2,004,099 - (115,368 + 905,197). This correction is fully in line with the correction for numerical inconsistency that must be made before the LS can be calibrated.

Table 16 Published transition matrix for 2018Q3-2018Q4, for age group 15-29, and comparison with distributions of ILO status based on quarterly calibrations

Age group 15-29							
2018Q3 \ 2018Q4	Unemployed current Q	Employed current Q	Inactive current Q	Total	*Estimate 2018Q3*	Differ-ence	% Differ-ence
Unemployed previous Q	40,128	42,782	33,796	116,706	***115,368***	1,338	1.16%
Employed previous Q	22,648	762,896	103,105	888,649	***905,197***	-16,548	-1.83%
Inactive previous Q	29,455	80,629	888,660	998,745	**-**	-	-
Total	92,231	886,307	1,025,561	2,004,099			
Estimate 2018Q4	***93,115***	***915,015***	***1,023,199***	**2,031,329**			
Difference	-844	-28,708	2,362	-27,230			
% Difference	-0,95%	-3,14%	0,23%	-1,34%			

It will be clear to the reader that similar inconsistencies between the column and row totals of transition matrices and the corresponding quarterly estimates of the distribution of ILO status can always occur – to a greater or lesser extent – if the sub-population for which the transition matrix is determined does not match the calibration variables in the model for estimating transition matrices.

3.2 Annual transitions per quarter: transitions between the same quarters in two consecutive years

Statbel publishes annual transitions per quarter (or quarter-specific annual transitions) in the downloadable Excel file LFS_TRANSITION_ENG_JQ_P.xlsx. In Table 17 below, we reproduce the global transition matrix, with associated transition percentage and sample size matrix, for 15-74-year-olds, as found in lines 11 to 14 in worksheet 2018Q3-2019Q3 in the above-mentioned Excel file.

This table should of course be read in the same way as Table 13.

Table 17 Published quarter-specific annual transition matrix for 2018Q3-2019Q3, with corresponding transition percentage and sample size matrices – see publication LFS_TRANSITION_ENG_JQ_P.xlsx

A. Transitions				
2018Q3 \ 2019Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	102,360	98,715	99,143	300,218
Employed previous Q	65,146	4,453,152	266,952	4,785,249
Inactive previous Q	106,414	295,994	2,962,820	3,365,228
B. Transition percentages				
2018Q3 \ 2019Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	34.1%	32.9%	33.0%	100.0%
Employed previous Q	1.4%	93.1%	5.6%	100.0%
Inactive previous Q	3.2%	8.8%	88.0%	100.0%
C. Unweighted transitions (respondent sample sizes)				
2018Q3 \ 2019Q3	Unemployed current Q	Employed current Q	Inactive current Q	Total
Unemployed previous Q	146	124	106	376
Employed previous Q	85	6,183	396	6,664
Inactive previous Q	116	365	4,510	4,991

Comparison of Table 13 and Table 17, in which each time 2018Q3 is the begin quarter, shows that the dynamics over a year is larger than over a quarter: while e.g. from quarter to quarter 50.1% remain unemployed, for a year this is 34.1%; for employed (94.8% vs. 93.21%) and inactive (91.6% vs. 88.0%) the difference is smaller, but the dynamics is still larger for the annual transitions.

Note that the results in Table 13 are based on an LS of 13,510 respondents, while the results in Table 17 are based on a slightly smaller LS of 12,031 respondents. This is of course due to a higher dropout rate over a year than over a quarter.

Section 3.1 illustrated that estimates of quarterly transitions for certain sub-populations (e.g. non-Belgians, or 55-64-year-olds) are based on few respondents, which makes the estimates inaccurate. The same applies, of course, to estimates of quarter-specific annual transitions.

Furthermore, it was also illustrated in section 3.1 that the marginals of quarterly transition matrices do not always reproduce the quarterly estimates for the distributions of ILO status in the begin quarter and end quarter, e.g. when the sub-population of 15-29-year-olds is isolated. This problem too arises in the same way for quarter-specific annual transition matrices.

3.3 Annual transitions: transitions between consecutive years

Finally, at the end of each calendar year Statbel publishes annual transitions in the downloadable Excel file LFS_TRANSITION_ENG_JJ_P.xlsx; currently, annual transitions are only available for 2017-2018, 2018-2019 and 2019-2020. To estimate the annual transitions, we calculate the unweighted average of four quarter-specific annual transitions, as explained in section 2.9. In other words, the annual transition matrices in LFS_TRANSITION_ENG_JJ_P.xlsx are unweighted averages of quarter-specific annual transition matrices in LFS_TRANSITION_ENG_JQ_P.xlsx. In Table 18 we show the annual transition matrix for 2018-2019, with corresponding transition percentage matrix and sample size matrix. Note that the annual transition percentage matrix is *not* the average of four quarter-specific annual transition percentage matrices, but is calculated directly from the annual transition matrix: see section 1.5.

Table 18 Published annual transition matrix for 2018-2019, with corresponding transition percentage and sample size matrices – see publication EAK_TRANSITIE_NL_JJ_P.xlsx

A. Transitions				
2019 / 2018	**Unemployed current J**	**Employed current J**	**Inactive current J**	**Total**
Unemployed previous J	109,689	90,340	100,714	300,743
Employed previous J	68,180	4,445,661	230,220	4,744,062
Inactive previous J	95,776	283,498	3,028,909	3,408,183
B. Transition percentages				
2019 / 2018	**Unemployed current J**	**Employed current J**	**Inactive current J**	**Total**
Unemployed previous J	36.5%	30.0%	33.5%	100.0%
Employed previous J	1.4%	93.7%	4.9%	100.0%
Inactive previous J	2.8%	8.3%	88.9%	100.0%
C. Unweighted transitions (respondent sample sizes)				
2019 / 2018	**Unemployed current J**	**Employed current J**	**Inactive current J**	**Total**
Unemployed previous J	602	465	496	1,563
Employed previous J	366	24,756	1,507	26,629
Inactive previous J	423	1,418	18,565	20,406

Panel B in Table 18 and panel B in Table 17 show the same tendency as regards the dynamics of the labour market situation from one year to the next (here: from 2018 to 2019): of the unemployed in 2018, around one-third (33.5%) are employed one year later, and around two-thirds are not (36.5% remain unemployed and 30.0% become inactive). It is of course clear from panels C in both tables that the annual transition (percentage) figures are more accurate than the quarter-specific annual transition (percentage) figures.

Table 19 below shows that for the sub-population of non-Belgians, for example, the annual transitions will be more accurate than the quarterly transitions (cf. Table 15) or the quarter-specific annual transitions for this sub-population. The same, of course, applies to other sub-populations. Consequently, breakdowns of annual transition matrices may go beyond breakdowns of quarter-specific transition matrices.

Table 19 Distribution of the longitudinal sub-sample 2018-2019 of non-Belgians (EU and Nt-EU combined) according to ILO status in begin and end quarter

2019 / 2018	Unemployed current J	Employed current J	Inactive current J	Total
Unemployed previous J	120	93	102	315
Employed previous J	69	2,414	149	2,632
Inactive previous J	90	161	1,793	2,044

The same remark as in sections 3.1 and 3.2 can also be made regarding the consistency between marginals of annual transition matrices and annual estimates for the distributions of ILO status in BQ and EQ.

3.4 A case study: transitions of short vs. long term unemployed

After a calibration is made for an LS, arbitrary sub-populations can be studied in more detail regarding their transitions. In this section, we illustrate this for the unemployed and compare the short-term and long-term unemployed.

Above, we made extensive use of the LS 2018Q3-2018Q4 to introduce Statbel's methods for estimating quarterly transitions. The estimated transition matrix is shown in Table 13. This section focuses exclusively on the unemployed in the BQ 2018Q3, and the effect of the duration of their unemployment on their transition probabilities. As usual, we differentiate between *short-term* unemployed – those who have been unemployed for one year or less (in 2018Q3) – and *long-term* unemployed – those who have been unemployed for at least one year (in 2018Q3). For a small number of unemployed in the BQ in the LS, the duration of unemployment is not known. Table 20 shows the estimated transitions for the unemployed in the BQ, broken down by duration of unemployment. Note that the rows labelled "All unemployed" in Table 20 correspond exactly to the rows labelled "Unemployed previous Q" in Table 13. The totals in the last column in panel A of Table 20 (except for the global total 300,216), which are sums of calibrated weights for unemployed respondents in the LS, do not equal corresponding estimates that could be made based on the quarterly sample for 2018Q3, for the simple reason that a differentiation by unemployment duration is not included in the calibration models.

Table 20 allows us to conclude that the long-term unemployed in 2018Q3 have a substantially higher likelihood (65.3%) of still being unemployed a quarter later than the short-term unemployed (37.3%). In a similar vein, in 2018Q3 short-term unemployed have a substantially higher likelihood (36.8%) of being employed a quarter later than long-term unemployed (13.1%). The transition probabilities for unemployed people with unknown unemployment duration should be ignored, due to the small number of respondents on which these estimates are based; we have added these for completeness, and to clarify the alignment with the results in Table 13.

Table 20 Quarterly transitions 2018Q3-2018Q4 for unemployed in the BQ, by duration of unemployment

A. Transitions for unemployed in BQ 2018Q3

2018Q3 \ 2018Q4	Unemployed	Employed	Inactive	Total
Short-term unemployed	60,551	59,826	42,018	162,395
Long-term unemployed	88,667	17,775	29,413	135,854
Employment duration unknown	1,237	729	-	1,967
All unemployed	150,455	78,330	71,431	300,216

B. Transition percentages for unemployed in BQ 2018Q3

2018Q3 \ 2018Q4	Unemployed	Employed	Inactive	Total
Short-term unemployed	37.3%	36.8%	25.9%	100.0%
Long-term unemployed	65.3%	13.1%	21.7%	100.0%
Employment duration unknown	62.9%	37.1%	-	100.0%
All unemployed	50.1%	26.1%	23.8%	100.0%

C. Unweighted transitions for unemployed in BQ 2018Q3

2018Q3 \ 2018Q4	Unemployed	Employed	Inactive	Total
Short-term unemployed	86	93	69	248
Long-term unemployed	137	25	61	223
Employment duration unknown	1	2	0	3
All unemployed	224	120	130	474

The same exercise can be done for annual transitions. Table 21 shows the estimated annual transition rates since the start of the Belgian panel survey for the unemployed in the begin year, broken down according to unemployment duration. Note that the row labelled "All unemployed" for begin year 2018 (2^{nd} panel in Table 21) corresponds exactly to the row labelled "Unemployed previous J" in panel B of Table 18.

Table 21 Annual transitions 2017-2018, 2018-2019 and 2019-2020 for unemployed in the begin year, by duration of unemployment

2017 \ 2018	Unemployed	Employed	Inactive	Total
Short-term unemployed	33.3%	41.3%	25.4%	100.0%
Long-term unemployed	51.3%	19.3%	29.4%	100.0%
Employment duration unknown	-	56.7%	43.3%	100.0%
All unemployed	42.1%	30.5%	27.5%	100.0%
2018 \ 2019	**Unemployed**	**Employed**	**Inactive**	**Total**
Short-term unemployed	27.4%	39.9%	32.7%	100.0%
Long-term unemployed	45.8%	19.9%	34.3%	100.0%
Employment duration unknown	-	51.7%	37.8%	100.0%
All unemployed	36.5%	30.0%	33.5%	100.0%
2019 \ 2020	**Unemployed**	**Employed**	**Inactive**	**Total**
Short-term unemployed	27.9%	36.9%	35.3%	100.0%
Long-term unemployed	52.9%	14.0%	33.1%	100.0%
Employment duration unknown	-	77.4%	22.6%	100.0%
All unemployed	38.9%	26.9%	34.2%	100.0%

This Table 21 too confirms the expected trends: the long-term unemployed have a higher (smaller) chance of being unemployed (employed) a year later than the short-term unemployed.

CONCLUSIONS

In this analysis, we have explained how the Belgian labour force transition matrices are produced by Statbel, and how they can be interpreted. We have shown that Eurostat's (2015b) method for producing transition matrices can be adapted to meet various requirements via a single model:

- Consistency with global distributions (for 15-74-year-olds) of ILO status in BQ and EQ;
- Consistency with the distributions of ILO status in BQ and EQ by sex;
- Consistency with distributions of ILO status in BQ and EQ for various other sub-populations;
- To a certain extent, transfer the structure of the calibrated EQ sample into the LS.

These sub-populations can be the cells in the crossing of two or more background variables, such as region, sex, age group, nationality category, education level, etc. In practice, of course, the size of the LS will be an important element in determining the categories of such variables, and in the choice of the variables to be included in the calibration model.

We have shown that known calibration techniques, as introduced by Deville and Särndal (1992), can be efficiently applied to make the calibrations for the production of transition matrices, with breakdowns into multiple background variables. The necessary preliminary corrections of distributions of ILO status in the BQ (by adjusting estimated numbers of inactive persons), in order to achieve consistency between the distributions of ILO status in BQ and EQ which are used as benchmarks in the calibration of the LS, can also be made using the same calibration techniques.

The SAS® macro CALMAR2 (Le Guennec and Sautory, 2002; Sautory, 1993), supplemented by additional macros that allow for efficient and flexible construction of CALMAR2 inputs, allows Statbel to easily apply sophisticated models. This facilitates to a significant extent the comparison of various candidate calibration models and the determination of a final model. Of course, the same can be achieved with many other software packages that have implemented the generally known calibration techniques of Deville and Särndal (1992).

Statbel's models for producing transition matrices are based on calibration of the LS, i.e. on the re-weighting of microdata. This is a fundamental difference from the original techniques proposed in Eurostat (2015b), where aggregate data are corrected each time. With the appropriate software, it is therefore easier to include more background variables into the models.

Another important difference with Eurostat (2015b) is that all desired corrections of the LS can be made simultaneously: corrections aimed at preserving distributions of ILO status in BQ and EQ, as well as corrections aimed at transferring to some extent the structure of the calibrated EQ sample into the LS.

From 2021 onward, Statbel applies these integrated methods for the production and publication of estimated transitions on the labour market.

Finally, we note that Statbel's solution for calibrating the LS can be an alternative to the application of econometric models (Kiiver and Espelage, 2016). Subject to further development of suitable variance estimation methods for evaluating the accuracy of the estimated transitions as well as for comparing transitions in sub-populations, estimating transition matrices for various sub-populations can immediately support statistical analysis (such as comparing sub-populations).

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ANNEXES

A OVERVIEW OF LONGITUDINAL SAMPLES

Table B 1 Composition of the longitudinal sample (LS) for pairs of consecutive quarters

Begin quarter	End quarter	RGs in the overlap	1st RG	2nd RG	3rd RG
			Observations from waves…		
2016Q3	2016Q4	1 and 2	1 and 2	1 and 2	-
2016Q4	2017Q1	1, 2 and 6	2 and 3	2 and 3	1 and 2
2017Q1	2017Q2	2 and 7	3 and 4	1 and 2	-
2017Q2	2017Q3	3, 4 and 8	2 and 3	2 and 3	1 and 2
2017Q3	2017Q4	5 and 9	2 and 3	1 and 2	-
2017Q4	2018Q1	6 and 10	3 and 4	1 and 2	-
2018Q1	2018Q2	7 and 11	3 and 4	1 and 2	-
2018Q2	2018Q3	8 and 12	3 and 4	1 and 2	-
2018Q3	2018Q4	9 and 13	3 and 4	1 and 2	-
2018Q4	2019Q1	10 and 14	3 and 4	1 and 2	-
2019Q1	2019Q2	11 and 15	3 and 4	1 and 2	-
2019Q2	2019Q3	12 and 16	3 and 4	1 and 2	-
2019Q3	2019Q4	13 and 17	3 and 4	1 and 2	-
2019Q4	2020Q1	14 and 18	3 and 4	1 and 2	-
2020Q1	2020Q2	15 and 19	3 and 4	1 and 2	-
2020Q2	2020Q3	16 and 20	3 and 4	1 and 2	-
2020Q3	2020Q4	17 and 21	3 and 4	1 and 2	-

Table B 2 Composition of the longitudinal sample (LS) for pairs of the same quarters in consecutive years

Begin quarter	End quarter	RGs in the overlap	1st RG	2nd RG
			Observations from waves…	
2016Q3	2017Q3	3	1 and 3	-
2016Q4	2017Q4	5 and 6	1 and 3	1 and 3
2017Q1	2018Q1	6 and 7	2 and 4	1 and 3
2017Q2	2018Q2	7 and 8	2 and 4	1 and 3
2017Q3	2018Q3	8 and 9	2 and 4	1 and 3
2017Q4	2018Q4	9 and 10	2 and 4	1 and 3
2018Q1	2019Q1	10 and 11	2 and 4	1 and 3
2018Q2	2019Q2	11 and 12	2 and 4	1 and 3
2018Q3	2019Q3	12 and 13	2 and 4	1 and 3
2018Q4	2019Q4	13 and 14	2 and 4	1 and 3
2019Q1	2020Q1	14 and 15	2 and 4	1 and 3
2019Q2	2020Q2	15 and 16	2 and 4	1 and 3
2019Q3	2020Q3	16 and 17	2 and 4	1 and 3
2019Q4	2020Q4	17 and 18	2 and 4	1 and 3

B GENERAL PRINCIPLES AND TERMINOLOGY OF CALIBRATION

This annex provides a brief introduction to calibration. It is in no way intended to be complete. The sole aim is to provide a better understanding of the main text (chapter 2), especially by clarifying the terminology of calibration theory. The **terms written in bold** below are (statistical) terms used (usually frequently) in the main text. These terms are written in bold in this annex so that the reader (of the main text) can easily find them here, in order to better understand their meaning. *Terms in italics* in this annex do not appear in the main text but are terms from calibration theory or from the underlying theory of mathematical optimisation; these terms are shown in italics only once.

More detail on calibration theory and its applications can be found in the literature; see section B.7 in this annex for a selection of publications. Särndal (2007) and Devaud & Tillé (2019) contain numerous other references discussing theory and applications.

B.1 OBJECTIVE

Calibration is a step in the processing of data where **initial weights** of the units in a dataset are adjusted to certain **reference distributions**. The direct result of calibration are **correction factors**, which are applied to the data to be calibrated in order to arrive at **calibrated estimates** of certain **indicators**. **Calibration** is therefore an estimation method.

B.2 AVAILABLE DATA

The data to be calibrated may or may not be aggregated. If not aggregated, then we assume that the data to be calibrated is a list of individual observations, where each observation consists of a set (or *vector*) of values for various variables, in which we distinguish three types: the **calibration variables**, the **study variables**, and the **initial weight**. The **indicators** referred to above are (combinations of) parameters of study variables, such as totals and averages, numbers and proportions, ratios, etc.

The most common example of non-aggregated data is the result of the observation of units in a random sample; the **initial weight** is then usually the **sampling weight**, but can also be an adjusted sampling weight, such as a sampling weight corrected for non-response, or a previously **calibrated weight**, etc.

In sections B.3 and B.4 in this annex B it is implicitly assumed that non-aggregated data should be calibrated. We will come back to aggregated data to be calibrated in section B.5.

The **reference distributions** referred to above are calculated from one or more datasets other than the one to be calibrated. The dataset(s) from which reference distributions must be determined may be in aggregate or non-aggregate form. In both cases, it can be a (study or target) population or a (weighted) sample. The reference distributions can be either true or estimated population distributions.

B.3 MATHEMATICAL OPTIMISATION PROBLEM

Calibration (of non-aggregated data) can always be formulated as a *mathematical optimisation problem*, more specifically as a (*mathematical*) *minimisation problem*. In the context of **calibration**, such a problem – which we will call the **calibration problem**, and later also the **calibration model** – consists of a set of **calibration equations**, and a *target function* (or *objective function*) to be minimised. The above-mentioned **correction factors** are the unknowns in the **calibration equations** and the arguments in the target function. Solving a calibration problem then amounts to solving the (mathematical) minimisation problem, i.e. finding a solution to the calibration equations that minimises the target function. We assume in this analysis that a solution can be found with the *method of Lagrange multipliers*.

B.3.1 CALIBRATION EQUATIONS

A **calibration equation** is a mathematical equality where the left-hand side is a weighted sum of a **calibration variable** over (a subset of) the dataset to be calibrated, and the right-hand side is a predetermined real number taken from the **reference distributions**. Each weight in the left-hand side is the product of a known **initial weight** and an unknown **correction factor**; we call these products the **calibrated weights**. The left-hand side can always be interpreted as an estimate of a certain parameter, the right-hand side as a reference value (true or previously estimated value) for that parameter. Consequently, the **calibration equations** determine which estimates, based on the dataset to be calibrated and using the **calibrated weights**, should be equal to certain reference values.

Calibration equations are always linear in the (unknown) **correction factors** as well as in the (unknown) **calibrated weights**.

B.3.2 DISTANCE MEASURE, CALIBRATION FUNCTION AND CALIBRATION METHOD

The objective function in the minimisation problem is a *global distance measure*, namely the sum of the *distances* $G_i(w_i, d_i)$ between the initial weight d_i and the calibrated weight w_i of the individual observations i; the statistician wishing to perform a calibration needs to choose the distances (via the software he or she wishes to use). Provided that the function $G_i(.,d_i)$, for each i, satisfies certain conditions (continuous differentiability, convexity, etc.), inverting the derivative of this function leads to a so-called *calibration function* $F_i(.)$, such that $w_i = d_i F_i(\mathbf{x}_i^T \boldsymbol{\lambda})$. Herein is $\mathbf{x}_i^T$ the row vector of calibration variables for observation i, and $\boldsymbol{\lambda}$ the column vector of *Lagrange multipliers*. Using the calibration functions, the **calibration equations** can be written as a function of the Lagrange multipliers. Solving the minimisation problem therefore transforms into solving the system of equations as a function of the Lagrange multipliers. If the solution $\boldsymbol{\lambda}$ exists and is found, then the **correction factors** $F_i(\mathbf{x}_i^T \boldsymbol{\lambda})$, and consequently the **calibrated weights** $w_i = d_i F_i(\mathbf{x}_i^T \boldsymbol{\lambda})$ can be calculated.

Choosing the distance functions $G_i(.,d_i)$ is the same as choosing the calibration functions $F_i(.)$. This choice amounts to choosing the so-called **calibration method**. Depending on the nature of the chosen calibration functions, we refer to the linear, the exponential, … **calibration method**.

Certain quadratic distances $G_i(w_i, d_i)$ lead to the linear calibration functions $F_i(\mathbf{x}_i^T \boldsymbol{\lambda}) = 1 + q_i \mathbf{x}_i^T \boldsymbol{\lambda}$, with $q_i = F'_i(0)$. With this choice of distances $G_i(w_i, d_i)$ we say that the **linear (calibration) method** is chosen. If for example $F_i(\mathbf{x}_i^T \boldsymbol{\lambda}) = exp(q_i \mathbf{x}_i^T \boldsymbol{\lambda})$, then we refer to the **exponential** (or **multiplicative**) **method**; we refer to the literature (section B.7) for the distance functions corresponding to these calibration functions.

The **linear method** can be supplemented by limits on the **correction factors**: $L \leq F_i(\mathbf{x}_i^T \boldsymbol{\lambda}) \leq U$, in which the *lower limit* L and the *upper limit* U have to be chosen by the statistician; we then refer to the *truncated linear method*. If the **exponential method** is supplemented in some way by such limits, then we refer to the *logit method*. By choosing appropriate limits for the correction factors, it can be avoided that **correction factors**, and thus indirectly also the **calibrated weights**, are negative or assume extreme values.

Other choices for the distances, calibration functions or **calibration methods** are discussed in the literature; developers of calibration software always make a (limited) selection of calibration methods they implement.

B.3.3 LINEAR STRUCTURE

From what precedes in this annex it must be clear that a **calibration problem** or **calibration model** is completely determined by the choice of the **calibration variables** and the choice of the **calibration method**. The choice of the **calibration variables** determines the **calibration equations** and the expression $\mathbf{x}_i^T \boldsymbol{\lambda}$, which we will call the **linear structure** (of the calibration model). This **linear structure** can be formally represented. To introduce a handy notation for the **linear structure**, we assume that a dataset has to be calibrated according to the distributions of three categorical (qualitative) variables A, B and C. With the (additive) **linear structure** A + B + C, we aim to calibrate according to the *marginal distributions* of the variables A, B and C. With the **linear structure** A + B*C, we aim to calibrate to the marginal distribution of the variable A and the *joint distribution* of the variables B and C. With the **linear structure** A*C + B*C, we aim to calibrate to the joint distribution of the variables A and C and the joint distribution of the variables B and C. With the **linear structure** A*B*C, we aim to calibrate to the joint distribution of the variables A, B and C. Etc.

It will be clear to the reader that with (only) three categorical variables some other linear structures can be determined. The notation can of course also be extended to more categorical variables. Applicability of a **calibration model** with a certain **linear structure** naturally depends on the availability of the **reference distributions** induced by the **linear structure**.

For a **linear structure** such as A*C + B*C, we call A*C and B*C the **terms**. The **linear structure** A + B + C has three **terms**, namely A, B and C.

Since in the main text of this analysis the calibration variables are always categorical or qualitative, we will not discuss here **linear structures** that include *quantitative* **calibration variables**.

Choosing a **linear structure** determines which variables x_j in the vectors $\mathbf{x}_i^T = (\cdots x_{ij} \cdots)$ should be included. In general, each **term** in the **linear structure** will imply a set of multiple variables x_j, and each of these variables is a 0-1 or indicator variable. For example, if A is a **term** in a **linear structure**, then for each value a of A a 0-1 variable will have to be created, which has value 1 for observations i for which A = a, and value 0 for observations i for which A ≠ a. For example, if B*C is a **term** in a **linear structure**, then for each combination bc of B and C (i.e. each cell bc in the crossing of the variables B and C) a 0-1 variable will have to be created, which has value 1 for observations i for which B = b and C = c, and value 0 for observations i for which B ≠ b and/or C ≠ c.

B.3.4 EXISTENCE AND UNIQUENESS OF SOLUTIONS

With the appropriate choice of distance functions $G_i(.,d_i)$, the solution to a **calibration problem** or **calibration model** is unique. This is primarily due to the necessary convexity of the distance functions. More detail can be found in the literature (section B.7).

The existence of a solution (which ultimately results in **correction factors** and **calibrated weights**) is formally related to the existence of a solution (possibly several solutions) to the system of **calibration equations**, extended with the possible limitation of the **correction factors** that follows from the choice of the **calibration method**. We list a number of aspects that may or may not lead to the existence of a solution to the extended system of **calibration equations**:

(1) The **referencedistributions** must be *consistent*. In particular, this always means that all **reference distributions** must lead to exactly the same total (population) figure. More generally, it also means that if different **terms** in the **linear structure** define the same sub-population (as the union of one or more cells resulting from the terms), the corresponding **reference distributions** must result in the same (sub-population) figure. Inconsistency can occur when **reference distributions** are calculated from different sources.

(2) The system of **calibration equations** should not be *overdetermined*. By this we mean, for example (!) the following. If a **term** in the **linear structure** generates a particular cell to which a non-zero **calibration total** corresponds, then that cell must be represented in the dataset to be calibrated. Formally, we can say that for any **calibration equation** where the right-hand side is non-zero, the left-hand side must not be an "empty sum".

(3) Any interval $[L,U]$ that defines the limits of the **correction factors**, if any, should not be too narrow. It is possible that the system of **calibration equations** without the limits has solutions, but that these do not meet the limits determined by $[L,U]$. Calibration software rarely provides an option to automatically calculate a (minimum) interval $[L,U]$ (*ReGenesees*, developed by ISTAT is an exception, see Zardetto, 2015). In practice, the user will then start by choosing "reasonable" values for L and U and try to solve the **calibration problem** with these limits. If a solution exists, the interval $[L,U]$ may be narrowed, by increasing L and/or decreasing U; if no solution exists, the interval $[L,U]$ must be widened, by decreasing L and/or increasing U. After that, another attempt can be made to solve the **calibration problem** with these new limits. Looking for an "optimal" interval $[L,U]$ (i.e. an interval the statistician is satisfied with), is therefore an iterative (practically *trial and error*) process.

B.4 PRACTICAL PROPERTIES

B.4.1 HIERARCHICAL NATURE OF THE LINEAR STRUCTURE, AND DISTRIBUTIVITY

If a **term** in the **linear structure** of a **calibration model** implies the joint distribution of variables A, B and C, for example, then that term also implies the joint distributions of A and B, of A and C and of B and C, as well as the marginal distributions of A, B and C. **Reference distributions** automatically satisfy this property (if calculated from the same source). Therefore, a calibration cannot be made according to the joint distribution of e.g. two variables A and B with a given model, without calibrating to the marginal distributions of A and B. Conversely, it is of course possible to calibrate to e.g. the marginal distributions of a number of variables with a given **calibration model**, but not to certain joint distributions of two or more of those variables. It follows that, for example, the linear structure A*B*C can be written in numerous ways:

A*B*C

= A*B*C + A*B + A*C + B*C

= A*B*C + A*B + A*C + B*C + A + B + C

= A*B*C + A + B + C

= ...

Moreover, the **term** 1 can always be added, as for example in A*B*C + 1, or in A + B*C + 1; here 1 stands for a categorical variable that takes on only one value, which incidentally is translated into a variable x_j with value 1 for all observations.

Finally, we can also apply a distributivity rule in the formulation of **linear structures**, as for example in A*B + A*C = A*(B + C), or in A + B*(C*D + E) = A + B*C*D + B*E.

These conventions in the notation of linear structures make it easy and efficient to discuss and propose various alternative **models**, one of which may be an extension or simplification of the other, or some of whose **terms** are common but not others.

The distributivity rule can be used to factorise a **linear structure**, as for instance in A*B + A*C = A*(B + C) in which the **term** A is isolated, or in A*B*C + D*E*B*C = (A + D*E)*B*C in which the **term** B*C is isolated. This can lead to a so-called *stratification* of **calibration models**, and the application of a simpler calibration model (B + C in the 1st example; A + D*E in the 2nd example) in each *calibration stratum* (i.e. category of A in the 1st example; cell in the crossing B*C in the 2nd example) separately. This may be useful to calibrate large datasets (Vanderhoeft, 2001), to let the **calibration method** and/or the limitation of **correction factors** depend on the calibration strata, and even to let the **linear structure** depend on the calibration strata.

B.4.2 POST-STRATIFICATION MODELS

If the **linear structure** is the complete crossing of all the **calibration variables** in it, we refer to a **post-stratification model**. Examples are: A*B*C, B*C, A, and also the simplest model with **linear structure** 1. "*Post-stratification is a simple, well-known and widely used weighting technique*", as stated and illustrated in Bethlehem (2008, *transl.*). This technique results in a unique **correction factor** and/or (final) weight for each post-stratum. Embedded in the calibration theory, such a special solution is generally not the only possible solution of the system of **calibration equations**, but by using any **calibration method**, this solution is indeed the result.

Note that for a **post-stratification model**, the choice of **calibration method** does not affect the result (provided that the limit of the **correction factors** is suitably chosen, and if the **calibration totals** are positive, which is the case in standard applications).

B.4.3 THE SAME CORRECTION FACTOR FOR ALL OBSERVATIONS WITH THE SAME CALIBRATION VARIABLES

From section B.3.2 in this annex we can conclude that the **correction factors** $F_i(\mathbf{x}_i^T\boldsymbol{\lambda})$ are the same for all observations i with the same vector of calibration variables $\mathbf{x}_i^T$. This can be seen as a generalisation of the inherent property of the post-stratification technique; see previous section B.4.2.

B.4.4 POSITIVE CORRECTION FACTORS AND CALIBRATED WEIGHTS

Not every **calibration method** necessarily results in only positive **correction factors**. The **linear method** can sometimes produce negative **correction factors**. The exponential method always produces only positive **correction factors** (if a solution exists). The truncated linear method can, however, produce only positive **correction factors** (if a solution exists) with the appropriate choice of the limiting interval $[L, U]$.

Finally, if we assume that the **initial weights** are positive, then the sign of the **correction factors** determines the sign of the **calibrated weights**.

B.5 AGGREGATED DATA

The data to be calibrated do not always present themselves as a set of individual data. This may be the case when the person who is to perform the calibration has no right to handle the datasets containing individual data (e.g. because the privacy of individual respondents needs to be protected), or if a researcher wants to try a calibration exercise on a set of statistics published in aggregate form.

In Eurostat (2015b), data to be calibrated are always presented as tables of estimates – more specifically as transition matrices; see also section 2.8 in chapter 2 of this analysis. This is undoubtedly due to the choice of the **algorithm** to perform the calibration, namely *iterative proportional fitting* (IPF). This **algorithm** has indeed been developed in the context of adjusting aggregate data (e.g. 2-dimensional or multi-dimensional frequency tables); an important application can be found in Eurostat (2003).

B.6 SOFTWARE USED BY STATBEL

Statbel's statisticians and methodologists use SAS® Enterprise Guide® for processing and analysing data. For calibration, the SAS®-macro CALMAR2 (Sautory, 1993; LeGuennec & Sautory, 2002) is used.

The input for CALMAR2 is twofold: (1°) a dataset containing the data to be calibrated (usually from a respondent sample), and (2°) a dataset containing the calibration totals for a given calibration model. Preparing the first dataset is rather elementary, given the simple structure of this dataset and the flexibility that the macro CALMAR2 offers in this respect. The preparation of the second dataset is more complex, firstly because the structure of that dataset is not standard, and secondly because each calibration model implies a different dataset. Indeed, this dataset partly defines the calibration model that will be applied. For this reason, Statbel has developed additional generic macros that create the second input dataset (with calibration totals) for CALMAR2 from one or more simply structured datasets and that also extend the first input dataset (with the data to be calibrated) with variables that CALMAR2 uses in the calibration. One such generic macro is CountsK, which by repeated application calculates the calibration totals corresponding to the different terms in the linear structure of the calibration model and adds them to the second input dataset, and with each application adds an appropriate calibration variable to the first input dataset. Macro CountsK treats each term A*B*C*... which depends on one or more categorical calibration variables A, B, C, ... separately. An example:

%CountsK(frame=*Frame1*, wei=*WEI*, sample=*Dataset1*, varlst=*A B C*, lev=*E*, term=*ABC*, …)

calculates the calibration totals from the dataset *Frame1* according to the term A*B*C, which results from the variable list "*A B C*", in a calibration model, using a weight variable *WEI* in this dataset, and stores these totals in one record of the dataset *Marges_E*. This record is given the identifier cv_E_ABC, as a result of the values of the arguments lev= and term= when CountsK is called; "cv" stands for *calibration variable*. Moreover, the macro CountsK will add the calibration variable cv_E_ABC to *Dataset1*, which contains the data to be calibrated; cv_E_ABC is a numbering of the cells in the crossing A*B*C. Macro CountsK also checks that each in *Frame1* non-empty cell in the A*B*C crossing is represented in *Dataset1*. Additional arguments of CountsK govern the initialisation of the dataset Marges_E, the addition of a record and the closing of Marges_E, as well as the production of an overview or report of the (repeated) application of CountsK.

For handling calibration terms involving one quantitative and one or more categorical variables, Statbel has developed the macro TotalsK; finally, for the treatment of contrast constraints or comparisons, there is the macro Contrast1. The macros TotalsK and Contrast1 are not used for calibration of longitudinal samples in this analysis. A complete treatment of the macros CountsK, TotalsK and Contrast1 is of course beyond the scope of this analysis.

We note that *Frame1*, from which calibration totals are calculated, can be either a population or a sample, and both can be in aggregate or non-aggregate form. Appropriate weights for calculating the calibration totals are stored in the variable *WEI*. Finally, it is also important that both *Frame1* and *Dataset1* contain the variables A, B, C, ...; furthermore, both *Frame1* and *Dataset1* can have a very simple structure.

The set of macros CALMAR2, CountsK, ... allows Statbel to flexibly experiment with a wide range of calibration models, to finally evolve towards a final model.

B.7 REFERENCES ON CALIBRATION THEORY

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C NC CALIBRATION MODELS: MATHEMATICAL ASPECTS

The original approach in Eurostat (2015b) for obtaining numerical consistency between the calibrated BQ and EQ samples is cell-based. Here, "cell" refers to any combination of background variables such as sex, region of residence, level of education, age group, etc.; by extension, a "cell" is also any combination of background variables and ILO labour market status. Eurostat (2015b) merely uses the background variables sex and age group.

In this annex we show that the Eurostat method of adjusting the calibrated BQ sample to the calibrated EQ sample can be formulated as a calibration model. This makes it possible to adapt Eurostat's original cell-by-cell approach if (for example) a cell is empty for the BQ but not for the EQ.

C.1 NOTATION

Below, we limit ourselves to the two background variables SEX (sex) and REG (region of domicile; if necessary, we differentiate between REG1 and REG2 for BQ and EQ respectively). SEX can take on values $s = 1$ (male) and $s = 2$ (female); REG can take on values $r = 1$ (BRU), $r = 2$ (VLA) and $r = 3$ (WAL). ILO status STAT has three possible values $b = 1$ (unemployed), $b = 2$ (employed) and $b = 3$ (inactive); depending on the context STAT stands for STAT1 or STAT2, i.e. ILO status in BQ or EQ respectively. A “cell” is a combination sr of SEX and REG, and by extension a combination srb of SEX, REG and STAT. The index i is used to identify the respondents; with notations such as $i \in sr$ (i.e. "i element of cell sr", or "i in cell sr", ...) we indicate that respondent i belongs to the sub-sample of persons for which SEX = s and REG = r. Similarly, $i \in srb$ indicates that respondent i belongs to cell sr and has ILO status b, or in short that i belongs to cell srb. The context should show whether a respondent i belongs to the BQ sample, the EQ sample or the longitudinal sample.

The variables SEX and/or REG can easily be replaced by other background variables or expanded to three or more background variables.

The following quantities are essential in the mathematical explanation in this annex:

- w_i^{BQ} = calibrated weight for respondent i in the BQ sample
- w_i^{EQ} = calibrated weight for respondent i in the EQ sample
- $1_{i \in sr}^{BQ} = 1$ if respondent i belongs to the BQ sample and is in sr, 0 otherwise
- $1_{i \in sr}^{EQ} = 1$ if respondent i belongs to the EQ sample and is in sr, 0 otherwise

- $T_{sr}^{BQ} = \sum_{i \in BQ} w_i^{BQ} 1_{i \in sr}^{BQ}$ is the estimated number of persons in the population in cell sr in the BQ (the summation is over the entire BQ sample)
- $T_{sr}^{EQ} = \sum_{i \in EQ} w_i^{EQ} 1_{i \in sr}^{EQ}$ is the estimated number of persons in the population in cell sr in the EQ (the summation is over the entire EQ sample)

Extension of the notation is obvious, and leads to new quantities; e.g.:

- Change from sr to s, which among other things leads to T_s^{BQ}, i.e. the estimated number of persons in the population in cell s in the BQ.
- Change from sr to r, which among other things leads to T_r^{BQ}, i.e. the estimated number of persons in the population in cell r in the BQ.
- Change from sr to srb, which among other things leads to
  - T_{srb}^{BQ}, i.e. the estimated number of persons in the population in cell srb (or: in cell sr and with ILO status STAT1 = b) in the BQ;
  - $1_{i \in srb}^{BQ} = 1$ if respondent i belongs to the BQ sample, is in cell sr and has ILO status STAT1 = b (or is in cell srb for short), 0 otherwise;

- $1^{EQ}_{i\in srb} = 1$ if respondent i belongs to the EQ sample, is in cell sr and has ILO status STAT2 = b (or is in cell srb for short), 0 otherwise.

C.2 CLASSICAL METHOD (NC-C)

NC-C model SEX * REG1 – a post-stratification model – implies a system of (calibration) equations, more specifically: a calibration equation for each cell sr:

$$\sum g_i w_i^{BQ} 1^{BQ}_{i\in sr} = T^{EQ}_{sr}$$

Note that the summation in the left-hand side of this equation runs across the entire BQ sample; this is so for every calibration equation in this annex, as it concerns the calibration of the BQ sample (to the calibrated EQ sample). The variables or unknowns in this system are the correction factors g_i (for individuals i who belong to the BQ sample); a solution is sought with numerical methods.

Since $g_i = g_{sr}$, for all $i \in sr$ (calibration theory!) in the BQ sample, it follows that

$$g_i = g_{sr} = T^{EQ}_{sr} \Big/ \sum w_i^{BQ} 1^{BQ}_{i\in sr} = T^{EQ}_{sr} \Big/ T^{BQ}_{sr}$$

whereby it is assumed that $T^{BQ}_{sr} > 0$. If $T^{BQ}_{sr} = 0$ for one or more cells sr, while $T^{EQ}_{sr} > 0$, then NC-C model SEX * REG1 cannot be applied, i.e. the system of calibration equations has no solution. Cells sr for which $T^{BQ}_{sr} > 0$ and $T^{EQ}_{sr} = 0$ do not pose a problem in technical terms, and then for these cells $g_i = g_{sr} = 0$.

The alternative NC-C model SEX + REG1 implies the system of calibration equations:

$$\sum g_i w_i^{BQ} 1^{BQ}_{i\in s} = T^{EQ}_{s} \quad \text{for each } s$$
$$\sum g_i w_i^{BQ} 1^{BQ}_{i\in r} = T^{EQ}_{r} \quad \text{for each } r$$

We assume that this system has a solution. Calibration theory states that still $g_i = g_{sr}$ for all $i \in sr$, i.e. the correction factors are constant within each combination sr, but an expression in closed form as in the case of NC-C model SEX * REG1 cannot be found. The g_{sr} are then obtained by solving the system of calibration equations using iteration, after choosing a target function that measures a (kind of) quasi-distance between the initial weights w_i^{BQ} and the calibrated weights $g_i w_i^{BQ}$. That target function (or distance) should be minimised under the system of calibration equations. (For a given choice of target function, the iterative method can be reduced to IPF.)

C.3 EUROSTAT METHOD (NC-E)

NC-E model SEX * REG1 * STAT1 – likewise a post-stratification model – implies for each combination sr three calibration equations:

$$\sum g_i w_i^{BQ} 1^{BQ}_{i\in srb} = T^{BQ}_{srb} \qquad \text{for} \quad b = 1 \text{ and } 2$$
$$\sum g_i w_i^{BQ} 1^{BQ}_{i\in srb} = T^{BQ}_{srb} + \left(T^{EQ}_{sr} - T^{BQ}_{sr}\right) = \tilde{T}^{BQ}_{srb} \qquad \text{for} \quad b = 3$$

Again, we assume that the system of calibration equations (one equation for each combination srb) has a solution.

The equations for $b = 1$ (unemployed) and 2 (employed) result in

$$g_i = g_{srb} = T^{BQ}_{srb} \Big/ T^{BQ}_{srb} = 1$$

if i belongs to cell sr and has ILO status STAT1 = b in the BQ. Of course, provided that $T^{BQ}_{srb} \neq 0$.

Note that $T^{BQ}_{srb} = 0$ will generally occur if the BQ sample has no respondents $i \in srb$: in this case, no g_i needs to be determined for $i \in srb$; for such an empty cell srb it is of course not necessary to include a calibration equation in the system. The case $T^{BQ}_{srb} = 0$ could (exceptionally) also occur for a non-empty cell srb, namely if $w_i^{BQ} = 0$ for all $i \in srb$; in this case, the g_i can take on an arbitrary (constant) value. $T_{srb}^{BQ} < 0$ is excluded, because in the calibration of the BQ sample the initial weights are positive, and the calibration method is chosen so that the correction factors are non-negative, so that all $w_i^{BQ} \geq 0$.

The equation for $b = 3$ (inactive persons) results in

$$g_i = g_{srb} = \tilde{T}_{srb}^{BQ} \Big/ T_{srb}^{BQ} = \left(T_{srb}^{BQ} + \left(T_{sr}^{EQ} - T_{sr}^{BQ}\right)\right) \Big/ T_{srb}^{BQ} = 1 + \frac{T_{sr}^{EQ} - T_{sr}^{BQ}}{T_{srb}^{BQ}}$$

if i belongs to cell sr and has ILO status STAT1 = $b = 3$ in the BQ. Here, too, we suppose that $T_{srb}^{BQ} \neq 0$, and the same remarks as for the equations for $b = 1$ and $b = 2$ can be made.

As such, unemployed and employed persons in the BQ sample do not get a new calibrated weight, inactive persons in the BQ sample may get one if i belongs to a cell sr for which $T_{sr}^{EQ} \neq T_{sr}^{BQ}$.

It is important to note that the calibration total $\tilde{T}_{sr3}^{BQ}$ can be negative – meaning that the correction factor g_i can also be negative – i.e. for a cell sr for which $\tilde{T}_{sr3}^{BQ} = T_{sr3}^{BQ} + \left(T_{sr}^{EQ} - T_{sr}^{BQ}\right) < 0$ or $T_{sr3}^{BQ} < T_{sr}^{BQ} - T_{sr}^{EQ}$, i.e. when the estimated total number of people in cell sr decreases between BQ and EQ (i.e. $T_{sr}^{EQ} < T_{sr}^{BQ}$), and this decrease (in absolute value) is greater than the number of inactive persons T_{sr3}^{BQ} in cell sr in the BQ. Consequently, we find that Eurostat's method for achieving numerical consistency between the BQ and EQ samples can lead to calibration models with negative calibration totals. See section 2.5.4 in chapter 2 for an illustration.

The alternative NC-E model (SEX + REG1) * STAT1 implies for each s three calibration equations:

$$\sum g_i w_i^{BQ} 1_{i \in sb}^{BQ} = T_{sb}^{BQ} \qquad \text{for} \quad b = 1 \text{ and } 2$$
$$\sum g_i w_i^{BQ} 1_{i \in sb}^{BQ} = T_{sb}^{BQ} + \left(T_s^{EQ} - T_s^{BQ}\right) = \tilde{T}_{sb}^{BQ} \qquad \text{for} \quad b = 3$$

and for each r also three calibration equations:

$$\sum g_i w_i^{BQ} 1_{i \in rb}^{BQ} = T_{rb}^{BQ} \qquad \text{for} \quad b = 1 \text{ and } 2$$
$$\sum g_i w_i^{BQ} 1_{i \in rb}^{BQ} = T_{rb}^{BQ} + \left(T_r^{EQ} - T_r^{BQ}\right) = \tilde{T}_{rb}^{BQ} \qquad \text{for} \quad b = 3$$

Assuming that this system has a solution $g_i = g_{srb}$ ($i \in srb$), then it cannot (generally) be expressed in closed form, but can be found by applying a numerical algorithm that simultaneously gives a solution for the system and minimises a chosen target function, which again measures a quasi-distance between initial weights w_i^{BQ} and calibrated weights $g_i w_i^{BQ}$. For this, Statbel currently uses the SAS® macro CALMAR2.

Note that, for example, the equation $\sum g_i w_i^{BQ} 1_{i \in sb}^{BQ} = T_{sb}^{BQ}$ in NC-E model (SEX + REG1) * STAT1 is the sum over r of the equations $\sum g_i w_i^{BQ} 1_{i \in srb}^{BQ} = T_{srb}^{BQ}$ in NC-E model SEX * REG1 * STAT1, for each s and for $b = 1$ or $b = 2$. Similarly, the equation $\sum g_i w_i^{BQ} 1_{i \in rb}^{BQ} = \tilde{T}_{rb}^{BQ}$ in NC-E model (SEX + REG1) * STAT1 is the sum over s of the equations $\sum g_i w_i^{BQ} 1_{i \in srb}^{BQ} = \tilde{T}_{srb}^{BQ}$ in NC-E model SEX * REG1 * STAT1, for each r and for $b = 3$. In practical terms, this means that if the calibration totals for the "maximum" NC-E model (SEX * REG1 * …) * STAT1 are determined, by simple summations of these calibration totals, the calibration totals for any "more limited" NC-E model, such as for example (SEX + REG1 + ...) * STAT1, can be obtained. This also means that any negative calibration totals $\tilde{T}_{sr3}^{BQ}$ in the maximum model may change to non-negative totals $\tilde{T}_{s3}^{BQ}$ and/or $\tilde{T}_{r3}^{BQ}$ in the more limited model.

Negative calibration totals are not common in practice, when a sample has to be calibrated to (estimated) population distributions, as the latter are always expressed in non-negative numbers. However, the explanations above show that negative calibration totals can logically result if the Eurostat method is chosen to achieve numerical consistency between the BQ and EQ samples. The occurrence of negative calibration totals is of course related to the chosen NC-E model, and, as explained in section 2.6 in chapter 2, this choice is related to the objectives of the calibration of the LS, i.e. to the required coherence between the marginals of transition matrices and the ILO status distributions in the BQ and the EQ. CALMAR2 makes it possible to work with negative calibration totals, provided a suitable calibration method is chosen. The (truncated) linear and the logit method allow negative correction factors, and thus negative calibrated weights; while the exponential or raking ratio, and the sine hyperbolic method, do not. Consequently, we have to choose either the (truncated) linear or the logit method to apply NC-E models, and in order not to have to change the method depending on whether or not negative calibration totals are to be used.

As we (currently) see no reason to limit the correction factors to a given interval, we will ultimately always choose the linear method when applying NC-E models.

D PERTURBATION OF THE LS IN SMALL SUB-POPULATIONS, IN VIEW OF CONSISTENCY REQUIREMENTS

The fact that the LS, as an overlap of the BQ and EQ samples, only contains about 50% of the respondents from each of the latter samples may pose a problem for small sub-populations in terms of achieving the desired consistency between the marginals of the transition matrix for that sub-population and the quarterly distributions of ILO status. A technical intervention – a perturbation of the LS – was worked out in this regard. We illustrate this using the sub-population of 65-74-year-olds, for the pair of quarters 2018Q3-2018Q4.

Table B 3 shows the following:

- Column (1): the BQ sample contains 4,663 respondents in the age group 65-74, 2 among whom are unemployed (in 2018Q3); column (3): only 1 of those 2 unemployed belongs to the LS;
- Column (2): The estimated distribution of ILO status in the BQ, in terms of number of unemployed and number of employed, should be reproduced in the transition matrix for 65-74-year-olds (if the LS model contains the term AGE1*STAT1);
- Column (4): the EQ sample contains 4,781 respondents in the age group 65-74, among whom 1 is unemployed (in 2018Q4); column (6): this unemployed person is not retained in the LS;
- Column (5): The estimated distribution of ILO status in the EQ must be fully reproduced in the transition matrix for 65-74-year-olds (if the LS model contains the term AGE2*STAT2).

Table B 3 Distribution by ILO status of 65-74-year-old respondents in BQ and EQ sample – with estimated distributions of ILO status –, and of 65-74-year-old respondents in BQ and in EQ in the LS for 2018Q3-2018Q4

	BQ ~ 2018Q3			**EQ ~ 2018Q4**		
ILO status	*Number of resp. in the BQ sample* (1)	*Estimated distribution ILO status* (2)	*Number of resp. in the LS* (3)	*Number of resp. in the EQ sample* (4)	*Estimated distribution ILO status* (5)	*Number of resp. in the LS* (6)
Unemployed	2	529.69	1	1	50.08	0
Employed	195	47,611.51	79	209	46,172.90	86
Inactive	4,466	1,079,204.22	2,134	4,571	1,085,211.02	2,189
Total	4,663	1,127,345.42	2,214	4,781	1,131,434.00	2,275

Table B 4, panel "*Before perturbation*", shows the distribution of the LS for 65-74-year-old respondents in the EQ according to ILO status in BQ and EQ (i.e. the unweighted transition matrix for 65-74-year-olds in the EQ). With regard to reproducing the distribution of ILO status in the EQ, the absence of unemployed respondents in the EQ in this LS poses a problem: the quarterly estimate 50.08 for the number of unemployed in EQ, and consequently the distribution of ILO status in the EQ (column (5) in Table B 3) cannot be reproduced.

Table B 4 Distribution according to ILO status in BQ and EQ of respondents aged 65-74 in the EQ, in the LS for 2018Q3-2018Q4, before and after perturbation of the LS

	ILO status EQ (2018Q4)							
	Before perturbation				***After perturbation***			
ILO status BQ (2018Q3)	*Unemployed*	*Employed*	*Inactive*	*Total*	*Unemployed*	*Employed*	*Inactive*	*Total*
Unemployed	0	0	1	1	0	0	1	1
Employed	0	69	23	92	0	69	23	92
Inactive	0	17	2,165	2,182	1	17	2,164	2,182
Total	0	86	2,189	2,275	1	86	2,188	2,275

An obvious way to solve this problem is to modify the LS calibration model, e.g. by working with other age groups (e.g. by using variable AGE2 instead of AGE2, so that age group 65-74 merges into the larger group 55-74; see section 2.1). However, this method does not allow the predefined ILO status distributions (e.g. for 55-64- and 65-74-year-olds separately) to be reproduced. As such, in the context of calibration for estimating transition matrices, we choose an alternative approach: the calibration model is not changed, but we perform a (minimal) random perturbation of the LS sample.

The perturbation implemented by Statbel is as follows: one of the persons inactive in the EQ in the LS (there are 2,189) is selected at random and his or her ILO status in the EQ is changed from inactive to unemployed. One possible result of this can be seen in the "*After perturbation*" panel of Table B 4; in this case, a respondent who is also inactive in the BQ was selected by chance (but with high probability!), but this is not necessarily always the case. The sample modified in this way (for 65-74-year-olds) does allow the term AGE2*STAT2 to be used in the LS calibration model – provided, of course, that other sub-populations do not pose a problem either.

In Table B 5 we show the estimated transition matrix for the sub-population of 65-74-year-olds, after applying the final calibration model LS-4 (after applying NC-E-3a) to the whole LS for 2018Q3-2018Q4. Note that the structure of this matrix is the same as the structure of the LS in the "*After perturbation*" panel of Table B 4. The marginals of the transition matrix reproduce, as desired, the distributions of ILO status in BQ and EQ that were shown in Table B 3 (columns (2) and (5)).

Table B 5 Estimated transition matrix for 65-74-year-olds in the EQ, after application of the final calibration model to the LS for 2018Q3-2018Q4, and *after* perturbation of the LS

	ILO status EQ (2018Q4)			
ILO status EQ (2018Q3)	*Unemployed*	*Employed*	*Inactive*	*Total*
Unemployed	-	-	529.69	529.69
Employed	-	33,523.40	14,088.11	47,611.51
Inactive	50.08	12,649.50	1,070,593.24	1,083,292.82
Total	50.08	46,172.90	1,085,211.04	1,131,434.01

Note that for 2018Q3-2018Q4, applying model LS-3 (after NC-E-3a) also requires a perturbation, as models LS-3 and LS-4 imply the same consistency requirements.

At the time this analysis was completed, 27 separate calibrations of LSs had already been performed: for the estimation of quarterly transitions for 15 pairs (2017Q1-2017Q2 to 2020Q3-2020Q4) and for the estimation of quarter-specific annual transitions for 12 pairs (2017Q1-2018Q1 to 2019Q4-2020Q4). For 11 of these 27 pairs, we used the perturbation technique to apply the final calibration model (section 2.7). In 5 of these 11 cases, the ILO status in the EQ of one arbitrary respondent in the LS changed from inactive to unemployed; in 5 other cases, the ILO status in the BQ of one arbitrary respondent in the LS changed from inactive to unemployed; in 1 case, the ILO status in the EQ of one arbitrary respondent in the LS changed from inactive to unemployed and also the ILO status in the BQ of one other arbitrary respondent in the LS changed from inactive to unemployed.

It is clear that this perturbation can also be applied in other sub-populations if necessary and/or if another ILO status is not represented, possibly if a different calibration model – e.g. in view of other consistency requirements – has to be applied.

ABOUT STATBEL

Statbel, the Belgian statistical office, collects, produces and publishes objective and relevant figures on the Belgian economy, society and territory.

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